Discussion 2A

Note: There will be no sections covering this worksheet. But we highly recommend finishing the discussion worksheet on your own time. It will help you with the upcoming homework.Ask questions in office hours if you have any.

1. **Proving Inequality**
   For all positive integers $n \geq 1$, prove that

$$\frac{1}{3^1} + \frac{1}{3^2} + \ldots + \frac{1}{3^n} < \frac{1}{2}$$

2. **True/False**

   If $n > 0$ is a positive integer, and S is a set of distinct positive integers, all of which are less than or equal to n, then S has at most n elements.

3. **More squares**
   Prove the following statement: $\forall n \in \mathbb{N}, \exists m \in \mathbb{N}$ such that $\sum_{k=0}^{n} k^3 = m^2$

4. **Stable Marriage**

   Consider the set of men $M = \{1, 2, 3\}$ and the set of women $W = \{A, B, C\}$ with the following preferences.

Men	Women		
1	A	B	C
2	B	A	C
3	A	B	C

Women	Men		
A	2	1	3
B	1	2	3
C	1	2	3

   Run the male propose-and-reject algorithm on this example. How many days does it take and what is the resulting pairing? (Show your work)

5. **Propose-and-Reject Proofs**

   Prove the following statements about the traditional propose-and-reject algorithm.

   (a) In any execution of the algorithm, if a woman receives a proposal on day i, then she receives some proposal on every day thereafter until termination.

   (b) In any execution of the algorithm, if a woman receives no proposal on day i, then she receives no proposal on any previous day j, $1 \leq j < i$.

   (c) In any execution of the algorithm, there is at least one woman who only receives a single proposal (Hint: use the parts above!)