CS 70 Fall 2016 | Discrete Mathematics and Probability Theory Seshia and Walrand | DIS 7a

1. **Countability Basics**

   (a) Is $f : \mathbb{N} \to \mathbb{N}$, defined by $f(n) = n^2$ an injection (one-to-one)? Briefly justify.

   (b) Is $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = x^3 + 1$ a surjection (onto)? Briefly justify.

2. **Count it!**
   For each of the following collections, determine and briefly explain whether it is finite, countably infinite (like the natural numbers), or uncountably infinite (like the reals):

   (a) The integers which divide 8.

   (b) The integers which 8 divides.

   (c) The functions from $\mathbb{N}$ to $\mathbb{N}$.

   (d) Computer programs that halt.

   (e) Computer programs that always correctly tell if a program halts or not.

   (f) Numbers that are the roots of nonzero polynomials with integer coefficients.

   (g) The number of points in the unit square $[0,1] \times [0,1]$

   (h) Computer programs that correctly return the product of their two integer arguments

3. **Countable and Uncountable.**

   (a) Give a bijection from the real number interval $(1,\infty)$ to the real number interval $(0,1)$. (Notice the intervals are open.)

   (b) Given an $n \times n$ matrix A where the diagonal consist of alternating 1's and 0's starting from 1, $A[0,0] = 1$, describe a n length vector from $\{0,1\}^n$ that is not equal to a row in the matrix. (Hint: the all ones vector or the all zeros vector of length n could each be rows in the matrix.)

4. **Computability.**

   (a) The problem of determining whether a program halts in time 2^{n^2} on an input of size n is undecidable. (True or False.)

   (b) There is no computer program DEAD which takes a program P, an input, x, and a line number, n, and determines whether the nth line of code is executed when the program P is run on the input x. (True or False.)