

1. Probability Practice

- (a) If we put 5 math, 6 biology, 8 engineering, and 3 physics books on a bookshelf at random, what is the probability that all the math books are together?
- (b) A message source M of a digital communication system outputs a word of length 8 characters, with the characters drawn from the ternary alphabet $\{0, 1, 2\}$, and all such words are equally probable. What is the probability that M produces a word that looks like a byte (*i.e.*, no appearance of '2')?
- (c) If five numbers are selected at random from the set $\{1, 2, 3, \dots, 20\}$, what is the probability that their minimum is larger than 5? (A number can be chosen more than once.)

2. Unlikely events

- (a) Toss a fair coin x times. What is the probability that you never get heads?
- (b) Roll a fair die x times. What is the probability that you never roll a six?
- (c) Suppose your weekly local lottery has a winning chance of $1/10^6$. You buy lottery from them for x weeks in a row. What is the probability that you never win?
- (d) How large must x be so that you get a head with probability at least 0.9? Roll a 6 with probability at least 0.9? Win the lottery with probability at least 0.9?

3. Flippin' Coins

Suppose I have a biased coin, with outcomes H and T , with the probability of heads $Pr[H] = \frac{3}{4}$ and the probability of tails $Pr[T] = \frac{1}{4}$. Suppose I perform an experiment in which I toss the coin 3 times—an outcome of this experiment is (X_1, X_2, X_3) , where $X_i \in \{H, T\}$.

- (a) What is the *sample space* for my experiment?
- (b) Which of the following are examples of *events*? Select all that apply.
 - i. $\{(H, H, T), (H, H), (T)\}$
 - ii. $\{(T, H, H), (H, T, H), (H, H, T), (H, H, H)\}$
 - iii. $\{(T, T, T)\}$
 - iv. $\{(T, T, T), (H, H, H)\}$
 - v. $\{(T, H, T), (H, H, T)\}$
- (c) What is the probability of the outcome H, H, T ?
- (d) What is the probability of the event that my outcome has exactly two heads?

4. **Shooting Range**

You and your friend are at a shooting range. You ran out of bullets. Your friend still has two bullets left but magically lost his gun. Somehow you both agree to put the two bullets into your six-chambered revolver in successive order, spin the revolver, and then take turn shooting. Your first shot was a blank. You want your friend to shoot a blank too, should you spin the revolver again before you hand it to your friend?