1. **Markov Chains: Prove/Disprove**

    (a) There exists an irreducible, finite Markov Chain for which there exist initial distributions that converge to different distributions.

    (b) There exists an irreducible, aperiodic, finite Markov Chain for which $P(X_{n+1} = j|X_n = i) = 1$ or 0 for all i, j.

    (c) There exists an irreducible, non-aperiodic Markov Chain for which $P(X_{n+1} = j|X_n = i) \neq 1$ for all i, j.

    (d) For an irreducible, non-aperiodic Markov Chain, any initial distribution not equal to the invariant distribution does not converge to any distribution.

2. **Pokemon Craze**

    You and your friend are both trying to catch a Dratini. Unfortunately, you each can only attempt to catch one Dratini per day. Once you or your friend catch a Dratini, that person stops while the other person continues to try to catch a Dratini if they haven't already caught one. The probability an attempt at catching a Dratini is successful is p. What is the expected number of days this process takes, if you two both try to catch a Dratini every day? Solve this using a Markov Chain with three states. (What is this in terms of two Geometric random variables?) (Also, consider how this problem would change if instead of stopping, the first person kept on trying to catch a Dratini in case he could donate it to the other person).

3. **Continuous Intro**

    (a) Is $f(x) = 2x$ from $0 \leq x \leq 1$, 0 otherwise a valid pdf? Why or why not? Is it a valid cdf? Why or why not?

    (b) Calculate $E[X]$ and $Var(X)$ for X with pdf $f(x) = 1/l$ from $0 \leq x \leq l$, 0 otherwise.

    (c) Suppose X and Y are independent and have pdfs $f(x) = 2x$ from $0 \leq x \leq 1$, 0 otherwise and $f(y) = 1$ from $0 \leq x \leq 1$. What is their joint distribution?

    (d) Calculate $E[XY]$ for the above X and Y.

4. **Uniform Distribution** You have two spinners, each having a circumference of 10, with values in the range $[0, 10)$. If you spin both (independently) and let X be the position of the first spinner and Y be the position of the second spinner, what is the probability that $X \geq 5$, given that $Y \geq X$?