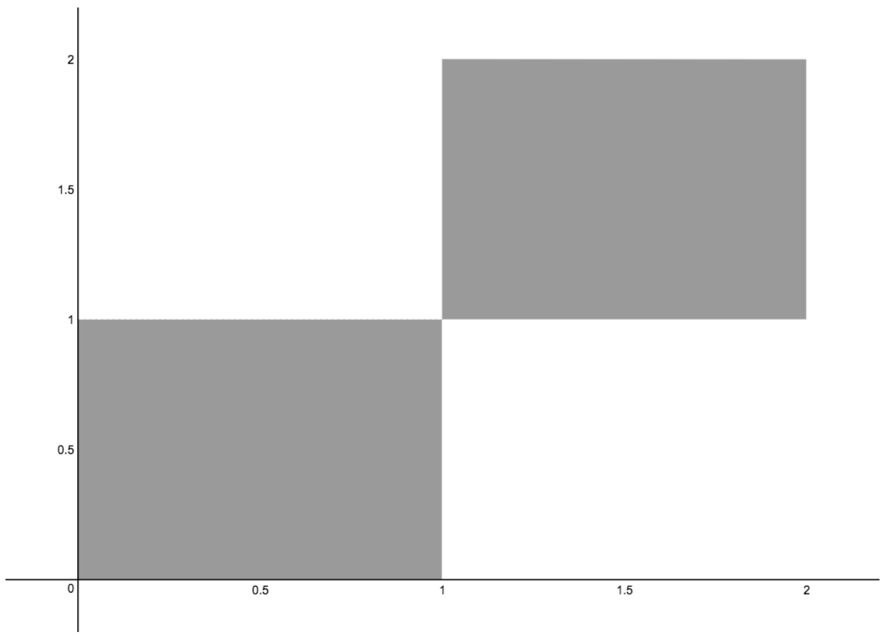
1. **Continuous LLSE**

Suppose that X and Y are uniformly distributed on the following figure:



That is, X and Y have the joint distribution

$$f_{X,Y}(x,y) = \begin{cases} 1/2, & 0 \le x \le 1, 0 \le y \le 1 \\ 1/2, & 1 \le x \le 2, 1 \le y \le 2 \end{cases}$$

(a) Do you expect X and Y to be positively correlated, negatively correlated, or neither?

(b) Compute the marginal distribution of X.

(c) Compute $L[Y \mid X]$.

(d) What is $E[Y \mid X]$?

2. **Conditioning on Exponentials**

Let X_i be i.i.d. Expo(λ) random variables.

(a) Compute $E[Y \mid Z]$, where $Y = \max\{X_1, X_2\}$ and $Z = \min\{X_1, X_2\}$.

(b) Compute $E[X_1 + X_2 \mid Z]$. (*Hint*: Use part (a).)

(c) Use part (b) to compute $E[Z]$.

(d) Compute $E[X_1 + X_2 \mid X_1 + X_2 + X_3]$.

3. **Erlang Distribution**

   In lecture, we proved the following **Fact**: if the lifetimes of light bulbs are i.i.d. $\text{Expo}(1)$ and we replace the light bulb as soon as one dies out, then the number of light bulbs we replace by time t follows the Poisson distribution with mean t. Using this fact, find the density of the sum of two i.i.d. $\text{Expo}(1)$ random variables.