CS 70　　　Discrete Mathematics and Probability Theory
Fall 2016　　Seshia and Walrand　　　　　　　　　　　　　**HW 14**

1　Sundry

Before you start your homework, write down your team. Who else did you work with on this homework? List names and email addresses. (In case of hw party, you can also just describe the group.) How did you work on this homework? Working in groups of 3-5 will earn credit for your "Sundry" grade.

Please copy the following statement and sign next to it:

I certify that all solutions are entirely in my words and that I have not looked at another student's solutions. I have credited all external sources in this write up.

2 Problems

1. **Strange Dilution**

   You have a jar of red marbles and blue marbles. At each time step, you draw a marble, and you note the color of the marble. Then, you dilute the proportion of the opposite-colored marbles by a factor of γ, where $0 < \gamma < 1$. (For example: if you pick a red marble, then the proportion of blue marbles is reduced by a factor of γ.) If p is the fraction of marbles that started off as red, what is the expected proportion of red marbles at time n?

2. **A meeting of three**

   James, Jon, and Mike each arrive at the movie theater at times uniformly distributed in the interval $(3:00, 3:20)$. Their arrival times are independent. Each person waits 5 minutes after their arrival before heading into the theater. What is the probability they all see each other before going into the theater?

3. **Exponential Median**

   What is the expected value of the median of three i.i.d exponential variables with parameter λ?

4. **Random walk**

   Alice starts at vertex 0 and wishes to get to vertex n. When she is at vertex 0 she has a probability of 1 of transitioning to vertex 1. For any other vertex i, there is a probability of $1/2$ of transitioning to $i+1$ and a probability of $1/2$ of transitioning to $i-1$. What is the expected number of steps Alice takes to reach vertex n?

5. **Exponential LLSE**

   Let $X \sim U[0,a]$ and let $Y = e^X$. Compute $L[Y \mid X]$. What does $L[Y \mid X]$ approach as $a \to 0$?

6. **First Exponential to Die**

   Let X and Y be $\text{Expo}(\lambda_1)$ and $\text{Expo}(\lambda_2)$ respectively. What is $P(\min(X,Y) = X)$, the probability that the first of the two to die is X?