70: Discrete Math and Probability Theory

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See note 1, for more discussion.

Instructors

Instructor: Sanjit Seshia.

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Taught: 149, 172, 144/244, 219C, EECS149.1x on edX, ...

Instructors

Jean Walrand – Prof. of EECS – UCB
257 Cory Hall – walrand@berkeley.edu

I was born in Belgium[(1)] and came to Berkeley for my PhD. I have been teaching at UCB since 1982.

My wife and I live in Berkeley. We have two daughters (UC alumni – Go Bears!). We like to ski and play tennis (both poorly). We enjoy classical music and jazz.

My research interests include stochastic systems, networks and game theory.

(1)

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Questions/Announcements $\implies$ piazza:
piazza.com/berkeley/fall2016/cs70

CS70: Lecture 1. Outline.

Today: Note 1.

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The language of proofs!

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The language of proofs!

1. Propositions.
2. Propositional Forms.
3. Implication.
4. Truth Tables
5. Quantifiers
6. More De Morgan's Laws

Propositions: Statements that are true or false.

$\sqrt{2}$ is irrational
2+2 = 4
2+2 = 3
826th digit of pi is 4
Jon Stewart is a good comedian
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$4 + 5$
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Put them together..

Propositions:

P_1 - Person 1 rides the bus.

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P_1 - Person 1 rides the bus.

P_2 - Person 2 rides the bus.

....

Suppose we can't have either of the following happen; That either person 1 or person 2 ride the bus and person 3 or 4 ride the bus. Or that person 2 or person 3 ride the bus and that either person 4 ride the bus or person 5 doesn't.

Propositional Form:

$\neg(((P_1 \vee P_2) \wedge (P_3 \vee P_4)) \vee ((P_2 \vee P_3) \wedge (P_4 \vee \neg P_5)))$

Who can ride the bus?

What combinations of people can ride the bus?

This seems ...complicated.

We need a way to keep track!

Truth Tables for Propositional Forms.

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T	T	T
T	F	
F	T	
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One use for truth tables: Logical Equivalence of propositional forms!

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Example: $\neg(P \wedge Q)$ logically equivalent to $\neg P \vee \neg Q$

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DeMorgan's Law's for Negation: distribute and flip!

$\neg(P \wedge Q)$

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$\neg(P \wedge Q) \equiv \neg P \vee \neg Q$ $\qquad$ $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

Implication.

$P \implies Q$ interpreted as

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If P, then Q.

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Example: Statement: If you stand in the rain, then you'll get wet.

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P = "you stand in the rain"

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Statement: "Stand in the rain"
Can conclude: "you'll get wet."

Non-Consequences/consequences of Implication

The statement "$P \implies Q$"

Non-Consequences/consequences of Implication

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only is False if P is True and Q is False .

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Some Fun: use propositional formulas to describe implication?

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Some Fun: use propositional formulas to describe implication?
$((P \implies Q) \wedge P) \implies Q$.

Implication and English.

$P \implies Q$

- If P, then Q.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.

Implication and English.

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- If P, then Q.
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Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
- P only if Q.
- P is sufficient for Q.

Implication and English.

$P \implies Q$

- If P, then Q.
- Q if P.
- P only if Q.
- P is sufficient for Q.
- Q is necessary for P.

Truth Table: implication.

P	Q	$P \implies Q$
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T	F	
F	T	
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Truth Table: implication.

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Truth Table: implication.

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P	Q	$\neg P \lor Q$
T	T	
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$\neg P \vee Q \quad \equiv P \implies Q$.

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$\neg P \vee Q \quad \equiv P \implies Q$.

These two propositional forms are logically equivalent!

Contrapositive, Converse

- Contrapositive of $P \implies Q$ is $\neg Q \implies \neg P$.

Contrapositive, Converse

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  - If the plant pollutes, fish die.

Contrapositive, Converse

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  - If the plant pollutes, fish die.
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- **Definition:** If $P \implies Q$ and $Q \implies P$ is P if and only if Q or $P \iff Q$.
  (Logically Equivalent: $\iff$.)

Variables.

Propositions?

- $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

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Next: Statements about boolean valued functions!!

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- See note 0 for more!

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"the square of a number is always non-negative"

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Quantifiers are not commutative.

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For False , find x, where $\neg P(x)$.

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Equivalent to:

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Equivalent to:

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English: means that for all x in S , $P(x)$ does not hold.

Which Theorem?

Theorem: $\forall n \in N \left(n \geq 3 \implies \neg(\exists a, b, c \in N \; a^n + b^n = c^n) \right)$

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Which Theorem?

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Next Time: proofs!