Today

Review for Midterm.

Propositional logic.

A proposition is a statement that is true or false.

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$3 = 4 - 1$?

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$3 = 4 - 1$? Proposition!

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Predicate: Statement with free variable(s).

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Example: $x = 3$

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$n > 3$?

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Predicate: Statement with free variable(s).

Example: $x = 3$ Given a value for x, becomes a proposition.

Predicate?

$n > 3$? Predicate: $P(n)$!

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Example: $x = 3$ Given a value for x, becomes a proposition.

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$x = y$?

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$(\forall n \in N), n^2 \geq n$. $(\forall x \in R)(\exists y \in R) y > x$.

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Connecting Propositions with Boolean Operators

$A \wedge B$, $A \vee B$, $\neg A$, $A \implies B$.

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Propositional Expressions and Logical Equivalence

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$

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Propositional Expressions and Logical Equivalence

$(A \implies B) \equiv (\neg A \vee B)$
$\neg(A \vee B) \equiv (\neg A \wedge \neg B)$

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$(A \implies B) \equiv (\neg A \vee B)$
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Proofs: truth table or manipulation of known formulas.

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Propositional Expressions and Logical Equivalence

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Proofs: truth table or manipulation of known formulas.

Boolean simplification rules - De Morgan's law, commutativity, associativity, etc.

Connecting Propositions with Boolean Operators

$A \land B$, $A \lor B$, $\neg A$, $A \implies B$.

Propositional Expressions and Logical Equivalence

$$(A \implies B) \equiv (\neg A \lor B)$$
$$\neg(A \lor B) \equiv (\neg A \land \neg B)$$

Proofs: truth table or manipulation of known formulas.

Boolean simplification rules - De Morgan's law, commutativity, associativity, etc.

$$(\forall x)(P(x) \land Q(x)) \equiv (\forall x)P(x) \land (\forall x)Q(x)$$

Proofs!

Direct: $P \implies Q$

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Example: a is even $\implies a^2$ is even.

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Direct: $P \implies Q$

Example: a is even $\implies a^2$ is even.

Approach: What is even?

Proofs!

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Approach: What is even? $a = 2k$

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What is even?

$a^2 = 2(2k^2)$

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Integers closed under multiplication!

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Contrapositive: $P \implies Q$

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Contrapositive: $P \implies Q$ or $\neg Q \implies \neg P$.

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Example: a^2 is odd $\implies a$ is odd.

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Example: a^2 is odd $\implies$ a is odd.

Contrapositive: a is even $\implies$ a^2 is even.

Contradiction: P

$\neg P \implies$ **false**

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$\neg P \implies R \wedge \neg R$

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Useful for prove something does not exist:

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Useful for prove something does not exist:

Example: rational representation of $\sqrt{2}$

Proofs!

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Useful for prove something does not exist:

Example: rational representation of $\sqrt{2}$ does not exist.

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Useful for prove something does not exist:

Example: rational representation of $\sqrt{2}$ does not exist.

Example: finite set of primes

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Example: rational representation of $\sqrt{2}$ does not exist.

Example: finite set of primes does not exist. Example: rogue couple does not exist.

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Induction.

$$P(0) \wedge ((\forall n)(P(n) \implies P(n+1) \equiv (\forall n \in N)\ P(n).$$

Induction.

$$P(0) \land ((\forall n)(P(n) \implies P(n+1) \equiv (\forall n \in N)\ P(n).$$

Thm: For all $n \geq 1$, $8 | 3^{2n} - 1$.

Induction.

$P(0) \wedge ((\forall n)(P(n) \implies P(n+1) \equiv (\forall n \in N)\ P(n).$

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Induction on n.

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Base: $8|3^2 - 1$.

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Induction on n.

Base: $8|3^2-1$.

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Thm: For all $n \geq 1$, $8 | 3^{2n} - 1$.

Induction on n.

Base: $8 | 3^2 - 1$.

Induction Hypothesis: True for some n.
$(3^{2n} - 1 = 8d)$

Induction.

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Thm: For all $n \geq 1$, $8 | 3^{2n} - 1$.

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Base: $8 | 3^2 - 1$.

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□

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n-men, n-women.

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No, for roommates problem.

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(Also called Traditional Marriage Algorithm)

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Not rogue couple!

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  Man optimal $\implies$ Woman pessimal.
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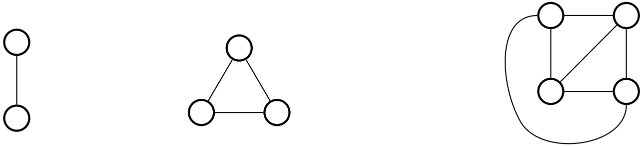
Graph Types: Complete Graph.



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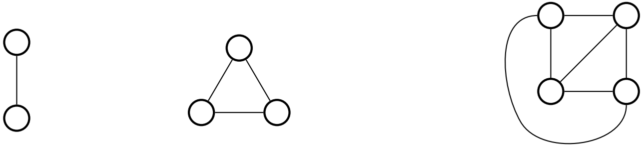

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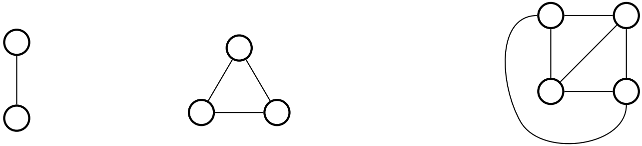

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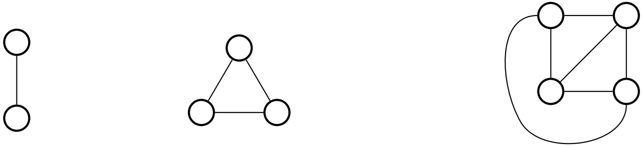

K_n, $|V| = n$

every edge present.
degree of vertex?

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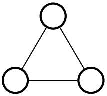

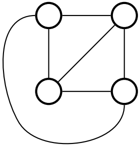

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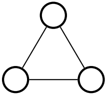

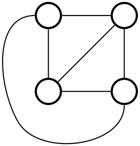

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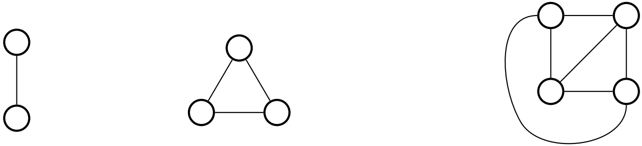

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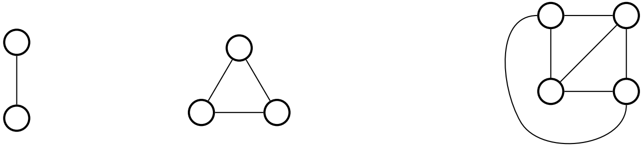

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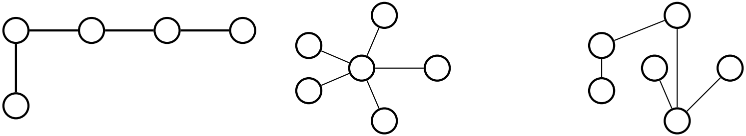
Very connected.
Lots of edges: $n(n-1)/2$.

Trees.



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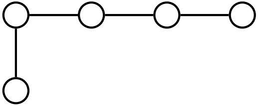
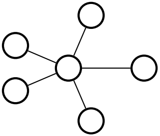
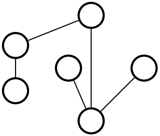

Definitions:

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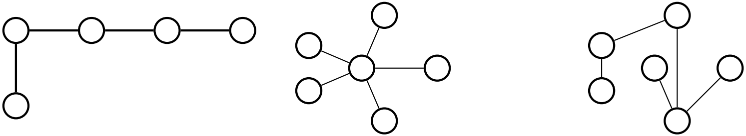

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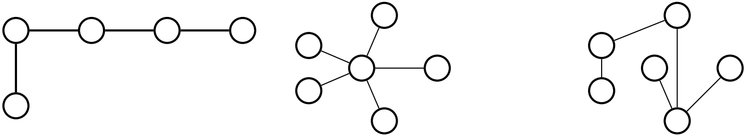

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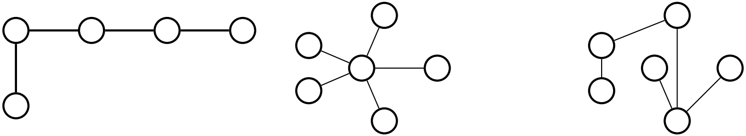

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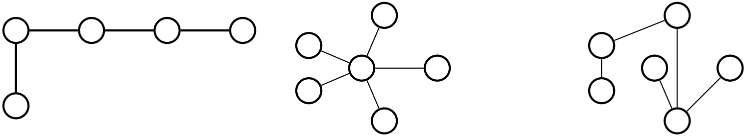

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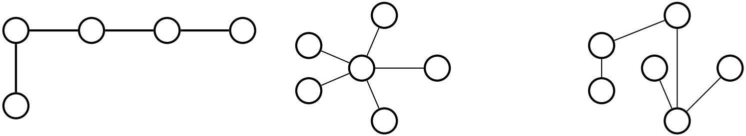

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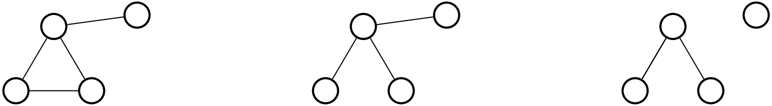

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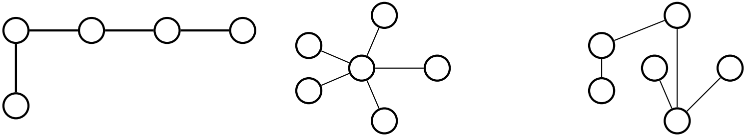

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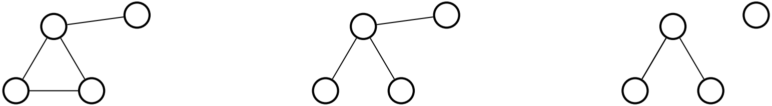

Minimally connected, minimum number of edges to connect.

Hypercube

Hypercubes.

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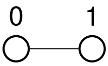

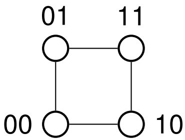

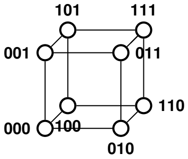

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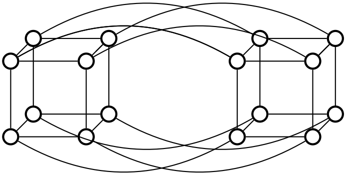

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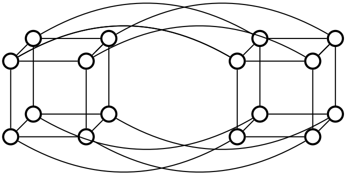

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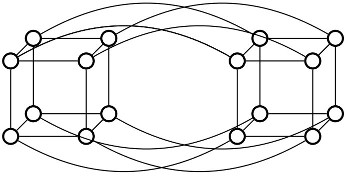

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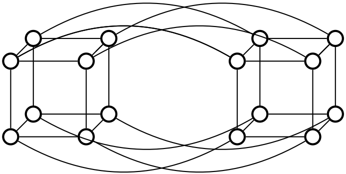

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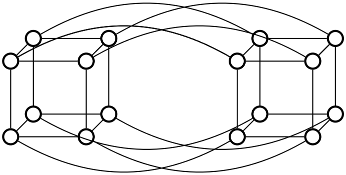

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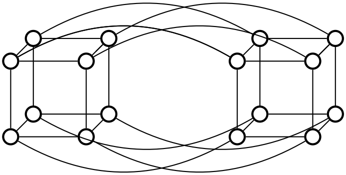

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Arithmetic modulo m.
  Elements of equivalence classes of integers.

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Time: 120 minutes.

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Many short answers.

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Get at ideas that we study.

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Midterm format

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