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Erasure Codes: Reconstructing a message if some parts of it (packets) are lost.

Idea: Encode n-packet message as a polynomial with n coefficients
Send values at $n+k$ points if $\leq k$ will be lost
Reconstruct from what you receive.

Today's topic.

Error Correction:

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Noisy Channel: corrupts k packets. (rather than loss/erasures.)

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Noisy Channel: corrupts k packets. (rather than loss/erasures.)

Additional Challenge: Finding which packets are corrupt.

Error Correction

Satellite

GPS device

Error Correction

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3 packet message.

GPS device

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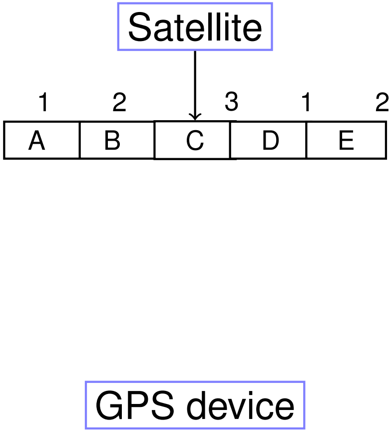
3 packet message.

Corrupts 1 packets.

GPS device

Error Correction



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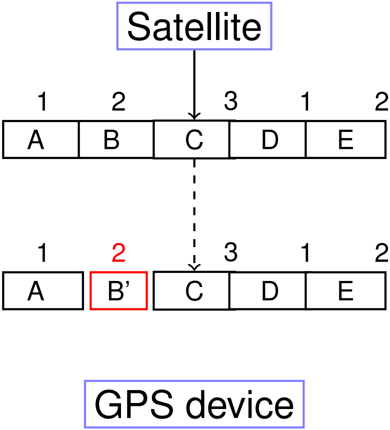

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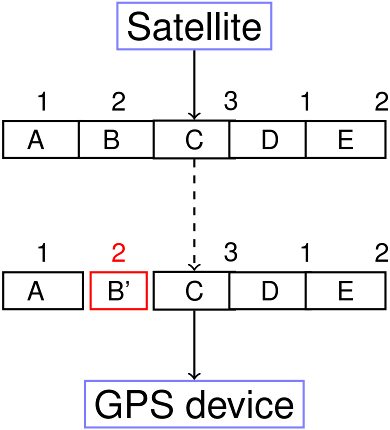

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$P(i) = R(i)$ for $n + k = 3 + 1 = 4$ points.

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Reconstructs $P(x)$ and only $P(x)$!!

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Assume point 1 is wrong

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\end{aligned}
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Assume point 1 is wrong and solve..

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

Find $P(x) = p_2 x^2 + p_1 x + p_0$ that contains $n + k = 3 + 1$ points.

All equations..

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\begin{aligned}
p_2 + p_1 + p_0 &\equiv 3 \pmod 7 \\
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Assume point 1 is wrong and solve..no consistent solution!
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In general..

$P(x) = p_{n-1}x^{n-1} + \cdots p_0$ and receive $R(1), \ldots R(m = n + 2k)$.

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Error!!

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Error!! .... Where???

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Something like $(n/k)^k$ ...Exponential in k!.

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How do we find where the bad packets are efficiently?!?!?!

Where can the bad packets be?

$$(p_{n-1} + \cdots p_0) \equiv R(1) \pmod{p}$$

Where can the bad packets be?

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
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Idea: Multiply equation i by 0 if and only if $P(i) \neq R(i)$.

Where can the bad packets be?

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We will use a polynomial!!!

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Error locator polynomial: $E(x) = (x - e_1)$

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$$\begin{aligned}(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p}\\ (p_{n-1}2^{n-1} + \cdots p_0) &\equiv R(2) \pmod{p}\\ &\vdots\\ (p_{n-1}(m)^{n-1} + \cdots p_0) &\equiv R(n+2k) \pmod{p}\end{aligned}$$

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$E(i) = 0$ if and only if $e_j = i$ for some j

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\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
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Multiply equations by $E(\cdot)$.

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Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

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(2-2)(4p_2 + 2p_1 + p_0) &\equiv (1)(2-2) \pmod 7 \\
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Multiply equation i by $(i - 2)$.

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Multiply equation i by $(i-2)$. All equations satisfied!

But don't know error locator polynomial!

Example.

Received $R(1)=3, R(2)=1, R(3)=6, R(4)=0, R(5)=3$

Find $P(x)=p_2x^2+p_1x+p_0$ that contains $n+k=3+1$ points.

Plugin points...

$$
\begin{aligned}
(1-2)(p_2+p_1+p_0) &\equiv (3)(1-2) \pmod 7\\
(2-2)(4p_2+2p_1+p_0) &\equiv (1)(2-2) \pmod 7\\
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4 unknowns (p_0, p_1, p_2 and e), 5 nonlinear equations.

The General Case.

$$
\begin{aligned}
(p_{n-1} + \cdots p_0) &\equiv R(1) \pmod{p} \\
&\vdots \\
(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i) \pmod{p} \\
&\vdots \\
(p_{n-1} m^{n-1} + \cdots p_0) &\equiv R(m) \pmod{p}
\end{aligned}
$$

The General Case.

$$
\begin{aligned}
E(1)(p_{n-1} + \cdots p_0) &\equiv R(1)E(1) \pmod{p} \\
&\vdots \\
E(i)(p_{n-1} i^{n-1} + \cdots p_0) &\equiv R(i)E(i) \pmod{p} \\
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Note: this is linear in a_i and coefficients of $E(x)$!

Finding $Q(x)$ and $E(x)$?

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$$Q(x) = a_{n+k-1}x^{n+k-1} + a_{n+k-2}x^{n+k-2} + \cdots a_0$$

Solving for $Q(x)$ and $E(x)$...

For all points $1, \ldots, i, n+2k$,

$$Q(i) = R(i)E(i) \pmod{p}$$

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Once we have those, compute $P(x)$ as $Q(x)/E(x)$.

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Solve for coefficients of $Q(x)$ and $E(x)$.

Once we have those, compute $P(x)$ as $Q(x)/E(x)$.

Example.

Received $R(1) = 3, R(2) = 1, R(3) = 6, R(4) = 0, R(5) = 3$

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6a_3 + 2a_2 + 3a_1 + a_0 &\equiv 6(3 - b_0) \pmod 7 \\
a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
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$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

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$Q(i) = R(i)E(i)$.

$$\begin{aligned}
a_3 + a_2 + a_1 + a_0 &\equiv 3(1 - b_0) \pmod 7 \\
a_3 + 4a_2 + 2a_1 + a_0 &\equiv 1(2 - b_0) \pmod 7 \\
6a_3 + 2a_2 + 3a_1 + a_0 &\equiv 6(3 - b_0) \pmod 7 \\
a_3 + 2a_2 + 4a_1 + a_0 &\equiv 0(4 - b_0) \pmod 7 \\
6a_3 + 4a_2 + 5a_1 + a_0 &\equiv 3(5 - b_0) \pmod 7
\end{aligned}$$

$a_3 = 1$, $a_2 = 6$, $a_1 = 6$, $a_0 = 5$ and $b_0 = 2$.

$Q(x) = x^3 + 6x^2 + 6x + 5$.

Example.

Received $R(1)=3, R(2)=1, R(3)=6, R(4)=0, R(5)=3$

$Q(x)=E(x)P(x)=a_3x^3+a_2x^2+a_1x+a_0$

$E(x)=x-b_0$

$Q(i)=R(i)E(i)$.

$$
\begin{aligned}
a_3+a_2+a_1+a_0 &\equiv 3(1-b_0) \pmod 7\\
a_3+4a_2+2a_1+a_0 &\equiv 1(2-b_0) \pmod 7\\
6a_3+2a_2+3a_1+a_0 &\equiv 6(3-b_0) \pmod 7\\
a_3+2a_2+4a_1+a_0 &\equiv 0(4-b_0) \pmod 7\\
6a_3+4a_2+5a_1+a_0 &\equiv 3(5-b_0) \pmod 7
\end{aligned}
$$

$a_3=1$, $a_2=6$, $a_1=6$, $a_0=5$ and $b_0=2$.

$Q(x)=x^3+6x^2+6x+5$.

$E(x)=x-2$.

Example: Compute $P(x)$.

$$Q(x) = x^3 + 6x^2 + 6x + 5.$$

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
           ------------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
```

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
```

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                1 x^2
       -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                1 x^2 + 6 x + 5
```

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                 1 x^2 + 1 x
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
```

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
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```
                1 x^2 + 1 x
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
```

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                1 x^2 + 1 x + 1
         -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
```

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          -----------------
    x - 2 ) x^3  + 6 x^2 + 6 x + 5
            x^3  - 2 x^2
            ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                           x + 5
                           x - 2
                           -----
                               0
```

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5.$
$E(x) = x - 2.$

```
                 1 x^2 + 1 x + 1
          -----------------
x - 2 )  x^3  + 6 x^2 + 6 x + 5
         x^3  - 2 x^2
         ----------
                 1 x^2 + 6 x + 5
                 1 x^2 - 2 x
                 ---------------
                         x + 5
                         x - 2
                         -----
                             0
```

$P(x) = x^2 + x + 1$

Example: Compute $P(x)$.

$Q(x) = x^3 + 6x^2 + 6x + 5$.
$E(x) = x - 2$.

```
                1 x^2 + 1 x + 1
        -----------------
x - 2 ) x^3  + 6 x^2 + 6 x + 5
        x^3  - 2 x^2
        ----------
                1 x^2 + 6 x + 5
                1 x^2 - 2 x
                ---------------
                          x + 5
                          x - 2
                          -----
                              0
```

$P(x) = x^2 + x + 1$
Message is $P(1) = 3, P(2) = 0, P(3) = 6$.

Error Correction: Berlekamp-Welch

Message: $m_1, \ldots, m_n$.

Error Correction: Berlekamp-Welch

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.

Error Correction: Berlekamp-Welch

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Error Correction: Berlekamp-Welch

Message: $m_1,\ldots,m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i)=m_i$.
2. Send $P(1),\ldots,P(n+2k)$.

Receiver:

1. Receive $R(1),\ldots,R(n+2k)$.

Error Correction: Berlekamp-Welch

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.

Error Correction: Berlekamp-Welch

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
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Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.

Error Correction: Berlekamp-Welch

Message: $m_1, \ldots, m_n$.
Sender:

1. Form degree $n-1$ polynomial $P(x)$ where $P(i) = m_i$.
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1. Receive $R(1), \ldots, R(n+2k)$.
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3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$,

Error Correction: Berlekamp-Welch

Message: $m_1, \ldots, m_n$.
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2. Send $P(1), \ldots, P(n+2k)$.

Receiver:

1. Receive $R(1), \ldots, R(n+2k)$.
2. Solve $n+2k$ equations, $Q(i) = E(i)R(i)$ to find $Q(x) = E(x)P(x)$ and $E(x)$.
3. Compute $P(x) = Q(x)/E(x)$.
4. Compute $P(1), \ldots, P(n)$, recover the message.

A key question.

Is there one and only one $P(x)$ from Berlekamp-Welch procedure?

A key question.

Is there one and only one $P(x)$ from Berlekamp-Welch procedure?

Existence: there is a $P(x)$ and $E(x)$ that satisfy equations.

Unique solution for $P(x)$?

Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \tag{1}$$

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Proof:

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We claim

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Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \tag{2}$$

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Equation 2 implies 1:

Unique solution for $P(x)$?

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Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$

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Uniqueness: any solution $Q'(x)$ and $E'(x)$ have

$$\frac{Q'(x)}{E'(x)} = \frac{Q(x)}{E(x)} = P(x). \quad (1)$$

Proof:
We claim

$$Q'(x)E(x) = Q(x)E'(x) \text{ on } n+2k \text{ values of } x. \quad (2)$$

Equation 2 implies 1:

$Q'(x)E(x)$ and $Q(x)E'(x)$ are degree $n+2k-1$
and agree on $n+2k$ points

Unique solution for $P(x)$?

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Cross divide.

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Cross divide. □

Revisiting last bit.

Claim: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

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Proof: Construction implies that

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$$Q'(i) = R(i)E'(i)$$

Revisiting last bit.

Claim: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
$$Q'(i) = R(i)E'(i)$$

for $i \in \{1, \ldots n+2k\}$.

Revisiting last bit.

Claim: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
$$Q'(i) = R(i)E'(i)$$

for $i \in \{1,\dots n+2k\}$.

If $E(i) = 0$, then $Q(i) = 0$.

Revisiting last bit.

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Proof: Construction implies that

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If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.

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$$Q(i) = R(i)E(i)$$
$$Q'(i) = R(i)E'(i)$$

for $i \in \{1, \ldots n+2k\}$.

If $E(i) = 0$, then $Q(i) = 0$. If $E'(i) = 0$, then $Q'(i) = 0$.
$\implies$ $Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

Revisiting last bit.

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When $E'(i)$ and $E(i)$ are not zero

Revisiting last bit.

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When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Revisiting last bit.

Claim: $Q'(x)E(x) = Q(x)E'(x)$ on $n+2k$ values of x.

Proof: Construction implies that

$$Q(i) = R(i)E(i)$$
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for $i \in \{1, \ldots n+2k\}$.

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When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points.

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$\implies Q(i)E'(i) = Q'(i)E(i)$ holds when $E(i)$ or $E'(i)$ are zero.

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$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points. $\square$

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When $E'(i)$ and $E(i)$ are not zero

$$\frac{Q'(i)}{E'(i)} = \frac{Q(i)}{E(i)} = R(i).$$

Cross multiplying gives equality in fact for these points. □

Points to polynomials, have to deal with zeros!

Berlekamp-Welch algorithm decodes correctly when at most k errors!

Summary. Error Correction.

Communicate n packets, with k erasures.

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How many packets?

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How many packets? $n+k$

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How many packets? $n+k$
How to encode?

Summary. Error Correction.

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree?

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How many packets? $n+k$
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Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Summary. Error Correction.

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How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
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How many packets? $n+2k$
How to encode?

Summary. Error Correction.

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Communicate n packets, with k errors.

How many packets? $n+2k$
How to encode? With polynomial, $P(x)$.

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
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Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
How to encode? With polynomial, $P(x)$. Of degree?

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
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Recover? Reconstruct $P(x)$ with any n points!

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How many packets? $n+2k$
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.
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Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
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Communicate n packets, with k errors.

How many packets? $n+2k$
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Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
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Communicate n packets, with k errors.

How many packets? $n+2k$
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.
Recover?
Reconstruct error polynomial, $E(x)$, and $P(x)$!

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
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Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.
Recover?
Reconstruct error polynomial, $E(x)$, and $P(x)$!
Nonlinear equations.

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
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Of degree? $n-1$
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Communicate n packets, with k errors.

How many packets? $n+2k$
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.
Recover?
Reconstruct error polynomial, $E(x)$, and $P(x)$!
Nonlinear equations.
Reconstruct $E(x)$ and $Q(x) = E(x)P(x)$.

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.
Recover?
Reconstruct error polynomial, $E(x)$, and $P(x)$!
Nonlinear equations.
Reconstruct $E(x)$ and $Q(x) = E(x)P(x)$. Linear Equations.

Summary. Error Correction.

Communicate n packets, with k erasures.

How many packets? $n+k$
How to encode? With polynomial, $P(x)$.
Of degree? $n-1$
Recover? Reconstruct $P(x)$ with any n points!

Communicate n packets, with k errors.

How many packets? $n+2k$
How to encode? With polynomial, $P(x)$. Of degree? $n-1$.
Recover?
Reconstruct error polynomial, $E(x)$, and $P(x)$!
Nonlinear equations.
Reconstruct $E(x)$ and $Q(x) = E(x)P(x)$. Linear Equations.
Polynomial division!

Summary. Error Correction.

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