Counting and Probability

What’s to come?

Counting and Probability

What's to come? Probability.

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What's to come? Probability.

A bag contains:

Counting and Probability

What's to come? Probability.

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What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Counting and Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue.

Counting and Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total.

Counting and Probability

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A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

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Today:

Counting and Probability

What's to come? Probability.

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Count blue. Count total. Divide.

Today: Counting!

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Today: Counting!

Later this week: Probability.

Counting and Probability

What's to come? Probability.

A bag contains:

What is the chance that a ball taken from the bag is blue?

Count blue. Count total. Divide.

Today: Counting!

Later this week: Probability. Professor Walrand.

Outline

1. Counting.
2. Tree
3. Rules of Counting
4. Sample with/without replacement where order does/doesn't matter.

Count?

How many outcomes possible for k coin tosses?
How many handshakes for n people?
How many 10 digit numbers?
How many 10 digit numbers without repeating digits?

Using a tree of possibilities...

How many 3-bit strings?

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How many different sequences of three bits from $\{0, 1\}$?

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How would you make one sequence?

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How many different ways to do that making?

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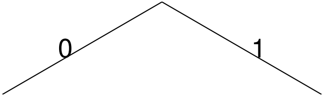

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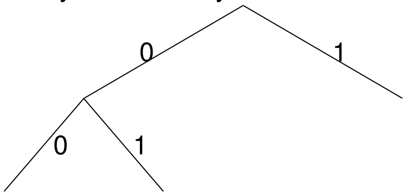

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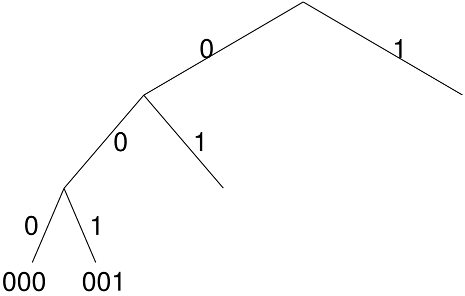

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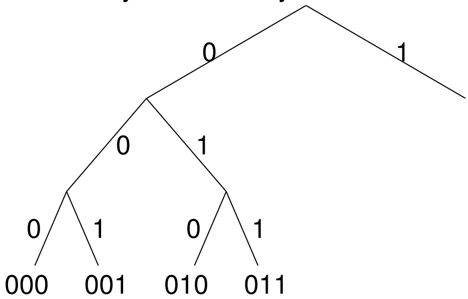

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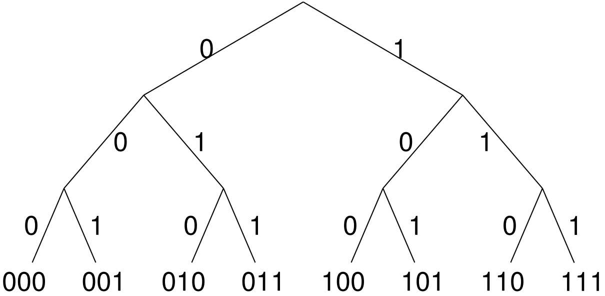

8 leaves which is $2 \times 2 \times 2$.

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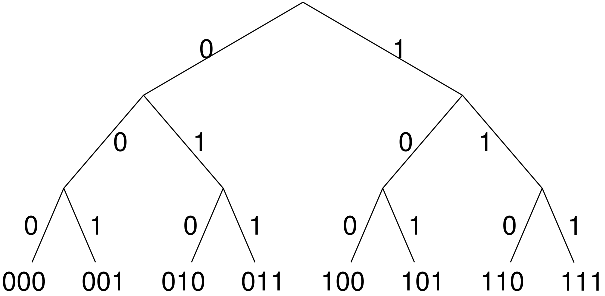

8 leaves which is $2 \times 2 \times 2$. One leaf for each string.

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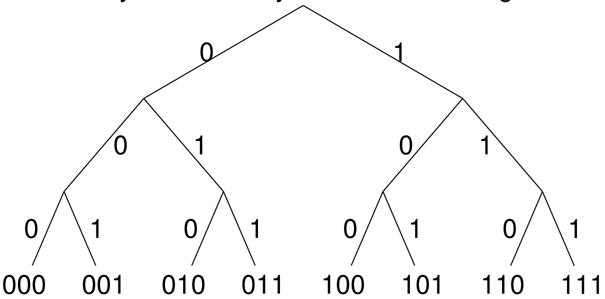

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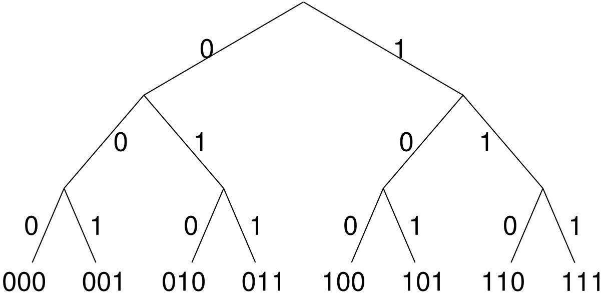

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8 3-bit srings!

First Rule of Counting: Product Rule

Objects made by choosing from n_1, then n_2, ..., then n_k
the number of objects is $n_1 \times n_2 \cdots \times n_k$.

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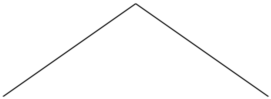

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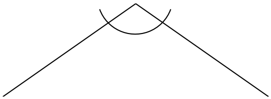

n_1

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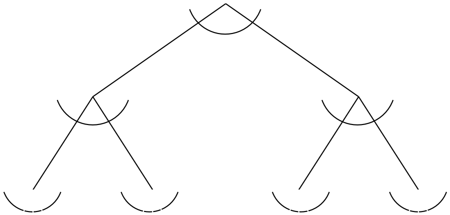

n_1

$\times n_2$

$\times n_3$

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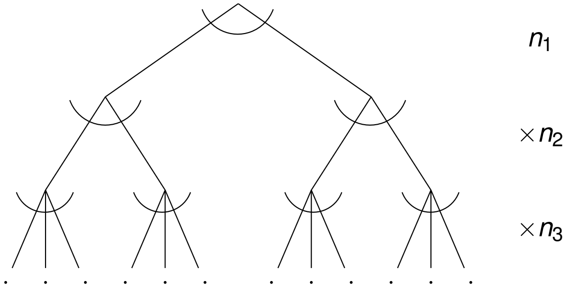

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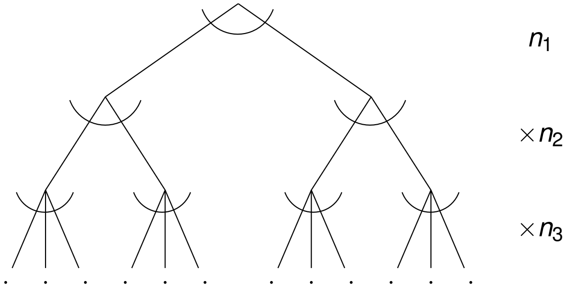

In picture, $2 \times 2 \times 3 = 12$

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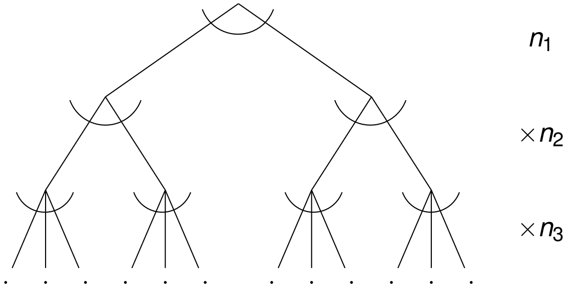

In picture, $2 \times 2 \times 3 = 12$

Using the first rule..

How many outcomes possible for k coin tosses?

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2 ways for first choice,

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2 ways for first choice, 2 ways for second choice, ...

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
2×2

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots$

Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
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Using the first rule..

How many outcomes possible for k coin tosses?

2 ways for first choice, 2 ways for second choice, ...
$2 \times 2 \cdots \times 2 = 2^k$

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$2 \times 2 \cdots \times 2 = 2^k$

How many 10 digit numbers?

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Functions, polynomials.

How many functions f mapping S to T?

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p ways to choose for first coefficient,

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Permutations.

[1] By definition: 0! = 1. $n! = n(n-1)(n-2)\ldots 1$.

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How many 10 digit numbers **without repeating a digit**?

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How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second,

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How many 10 digit numbers **without repeating a digit**?

10 ways for first, 9 ways for second, 8 ways for third,

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So total number is $|S| \times |S|-1 \cdots 1 = |S|!$
A one-to-one function is a permutation!

Counting sets..when order doesn't matter.

How many poker hands?

[2] When each unordered object corresponds equal numbers of ordered objects.

Counting sets..when order doesn't matter.

How many poker hands?

$52 \times 51 \times 50 \times 49 \times 48$

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and $10, J, Q, K, A$ of spades the same?

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Can write as...

$$\frac{52!}{5! \times 47!}$$

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$$\frac{52 \times 51 \times 50 \times 49 \times 48}{5!}$$

$$\frac{52!}{5! \times 47!}$$

Generic: ways to choose 5 out of 52 possibilities.

[2] When each unordered object corresponds equal numbers of ordered objects.

When order doesn’t matter.

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Choose 2 out of n?

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Notation: $\binom{n}{k}$ **and pronounced "**n **choose** k**."**

Simple Practice.

How many orderings of letters of CAT?
3 ways to choose first letter, 2 ways to choose second, 1 for last.
$\implies 3 \times 2 \times 1 = 3!$ orderings
How many orderings of the letters in ANAGRAM?
Ordered, except for A!
total orderings of 7 letters. 7!
total “extra counts” or orderings of two A’s? 3!

Total orderings? $\frac{7!}{3!}$
How many orderings of MISSISSIPPI?
4 S’s, 4 I’s, 2 P’s.
11 letters total!
11! ordered objects!
$4! \times 4! \times 2!$ ordered objects per “unordered object”

$$\implies \frac{11!}{4!4!2!}.$$

Sampling...

Sample k items out of n

Sampling...

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Without replacement:

Sampling...

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Without replacement:

Order matters:

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots$

Sampling...

Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1$

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Sample k items out of n

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

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Sample k items out of n

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Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders

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Sample k items out of n

Without replacement:

Order matters: $n \times n-1 \times n-2 \ldots \times n-k+1 = \frac{n!}{(n-k)!}$

Order does not matter:

Second Rule: divide by number of orders – "k!"

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Sampling...

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Second Rule: divide by number of orders – "k!"

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With Replacement.

Order matters: $n \times n$

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With Replacement.

Order matters: $n \times n \times \ldots n$

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Set: $1, 2, 3$

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Set: $1,2,2$

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How do we deal with this situation?!?!

Stars and bars....

How many ways can Bob and Alice split 5 dollars?

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For each of 5 dollars pick Bob or Alice(2^5), divide out order

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For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

5 dollars for Bob and 0 for Alice:
one ordered set: (B, B, B, B, B).

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How many ways can Bob and Alice split 5 dollars?
For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

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4 for Bob and 1 for Alice:

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5 ordered sets: (A, B, B, B, B) ; (B, A, B, B, B); ...

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Well, we can list the possibilities.
$0+5$, $1+4$,$2+3$, $3+2$, $4+1$, $5+0$.

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For each of 5 dollars pick Bob or Alice(2^5), divide out order ???

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For 3 numbers adding to k?

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How many ways to add up n numbers to equal k?

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Or: k choices from set of n possibilities with replacement.
Sample with replacement where order just doesn't matter.

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How many ways to add up n numbers to equal k?

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How many ways can Alice, Bob, and Eve split 5 dollars.

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Sample with replacement where order just doesn't matter.

How many ways can Alice, Bob, and Eve split 5 dollars.

Think of Five dollars as Five stars: $\star\star\star\star\star$.

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Alice: 2, Bob: 1, Eve: 2.

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Each split $\implies$ a sequence of stars and bars.

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Counting Rule: if there is a one-to-one mapping between two sets they have the same size!

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Answer: $|S| + |T| - |S \cap T| = 10^9 + 10^9 - 10^8$.

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