CS70: Sanjit Seshia & Jean Walrand: Lecture 24b.

Review M2 - Probability

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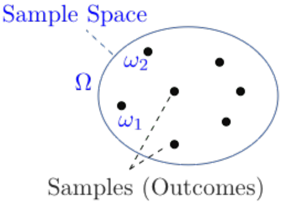
Probability: Midterm 2 Review.

- Framework:
  - Probability Space
  - Conditional Probability & Bayes' Rule
  - Independence
  - Mutual Independence
- Notes:
  - Note 25b: Page 1 + Bayes' Rule on page 2.
  - Note 13
  - Note 14

Review: Probability Space



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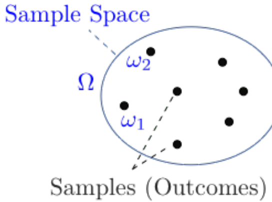

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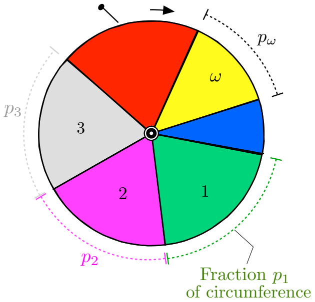

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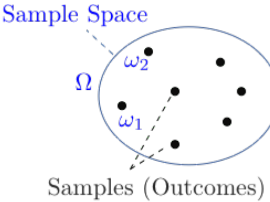

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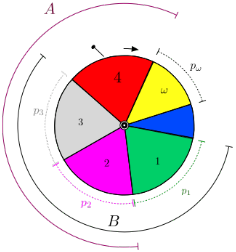

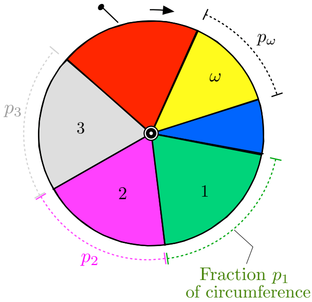

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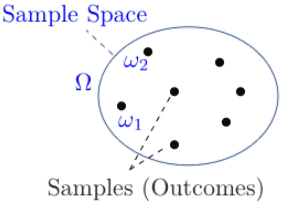

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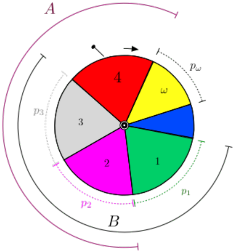

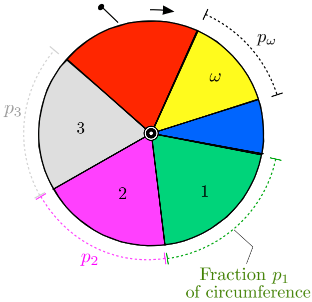

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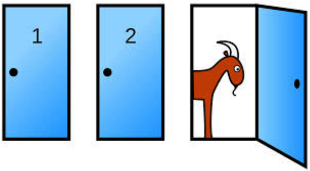

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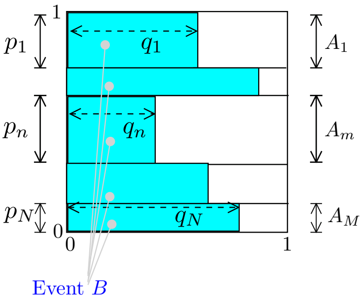

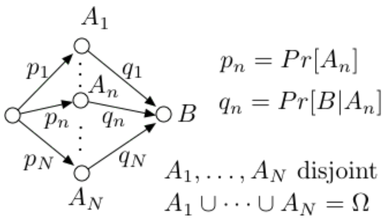

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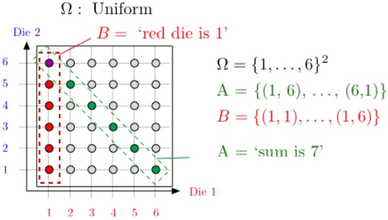

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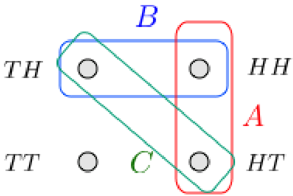

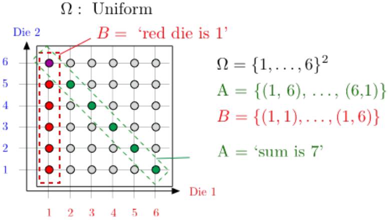

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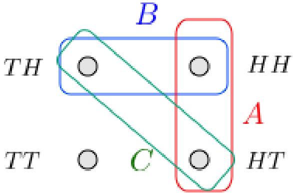

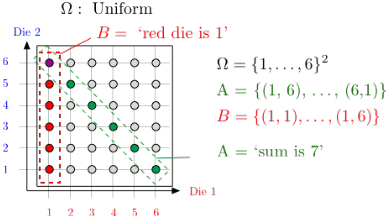

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Pairwise, but not mutually

If $\{A_j, i \in J\}$ are mutually independent, then $[A_1 \cap \bar{A}_2] \Delta A_3$ and $A_4 \setminus A_5$ are independent.

Our intuitive meaning of "independent events" is mutual independence.

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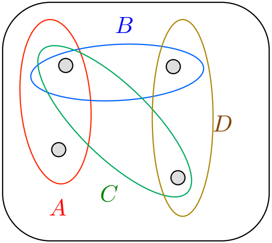

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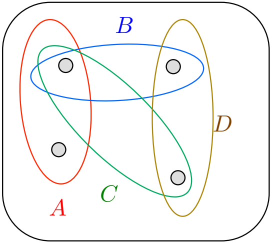

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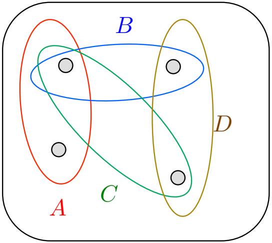

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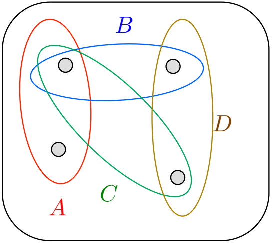

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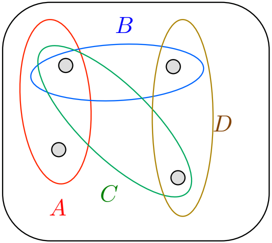

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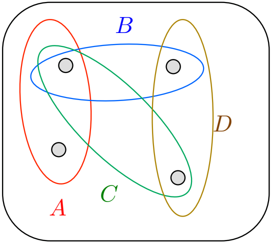

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No: We would need an outcome with probability $1/8$.

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This is not possible since $a, b < p$.

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An L^2-bounded martingale converges almost surely.

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$$Pr[A \cup B \cup C] \leq Pr[A] + Pr[B] + Pr[C]$$

Inclusion/Exclusion:

$$Pr[A \cup B] = Pr[A] + Pr[B] - Pr[A \cap B]$$

Total Probability:

$$Pr[B] = Pr[A_1]Pr[B|A_1] + \cdots + Pr[A_n]Pr[B|A_n]$$

An L^2-bounded martingale converges almost surely. Just kidding!

A mini-quizz

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- $Pr[A \cup B] = Pr[A] + Pr[B]$.

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- Deal two cards from a 52-card deck.

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  $Pr[\text{same}] = \frac{3}{51}$.

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  $Pr[\text{first} > \text{second}] = \frac{24}{51}$.