CS70: Jean Walrand: Lecture 35.

Continuous Probability 2

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Continuous Probability 2

1. Review: CDF, PDF
2. Examples
3. Properties
4. Expectation
5. Expectation of Function
6. Variance
7. Independent Continuous RVs

Review: CDF and PDF.

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Key idea:

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A Picture

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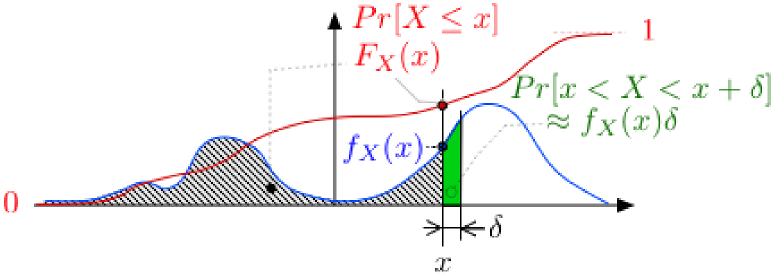

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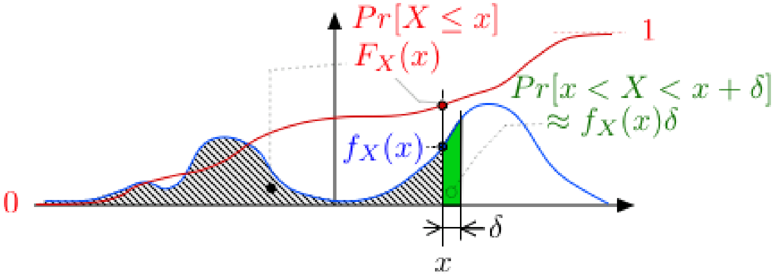

The pdf $f_X(x)$ is a nonnegative function that integrates to 1.

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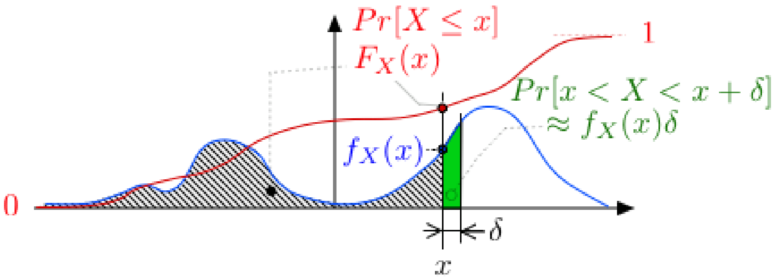

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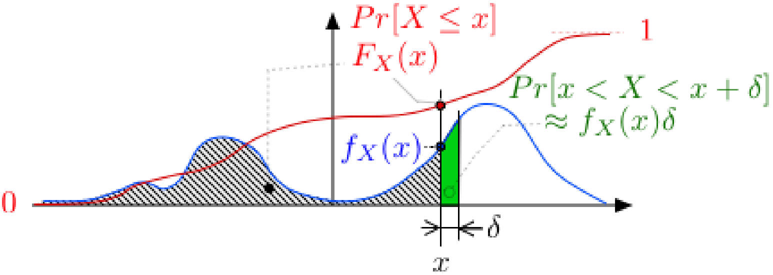

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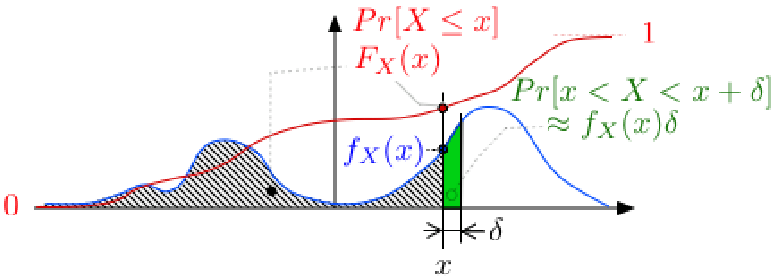

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$$Pr[x < X < x + \delta] \approx f_X(x)\delta$$

$$Pr[X \leq x] = F_x(x) = \int_{-\infty}^{x} f_X(u)du$$

Target

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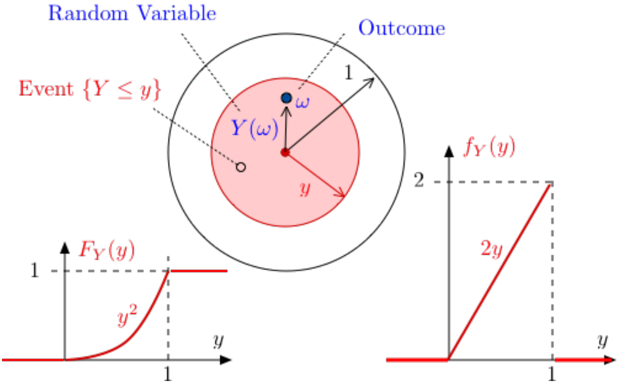

Random Variable
Outcome
Event $\{Y \leq y\}$
1
ω
$Y(\omega)$
y
$F_Y(y)$
1
y^2
y
1
$f_Y(y)$
2
$2y$
y
1

$U[a, b]$

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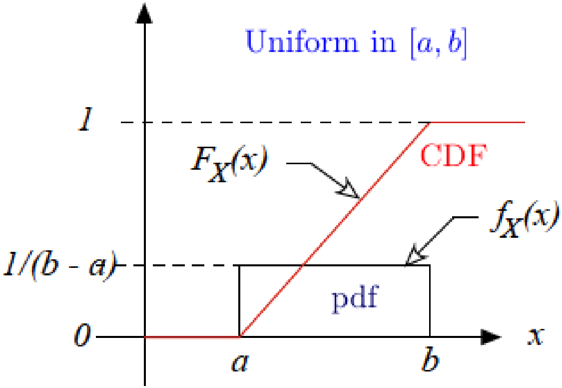

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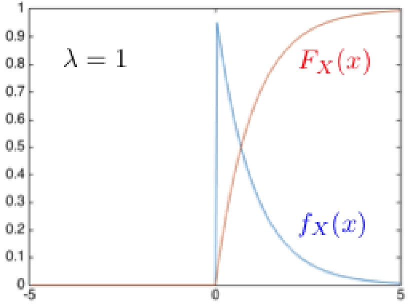

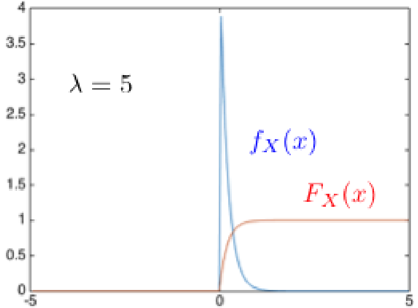

$Expo(\lambda)$

The exponential distribution with parameter $\lambda > 0$ is defined by

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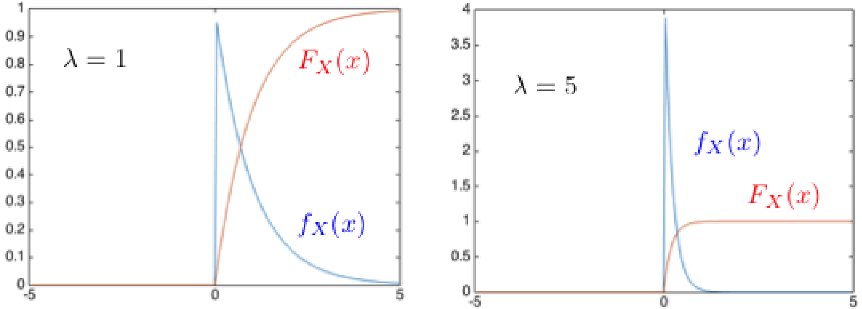

Note that $Pr[X > t] = e^{-\lambda t}$ for $t > 0$.

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\begin{aligned}
Pr[X > t + s \mid X > s] &= \frac{Pr[X > t + s]}{Pr[X > s]} \\
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Also, $Expo(\lambda) = \frac{1}{\lambda} Expo(1)$.

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$$\begin{aligned} Pr[Y \in (y, y+\delta)] &= Pr[a + bX \in (y, y+\delta)] = Pr[X \in (\frac{y-a}{b}, \frac{y+\delta-a}{b})] \\ &= Pr[X \in (\frac{y-a}{b}, \frac{y-a}{b} + \frac{\delta}{b})] = \frac{1}{b}\delta, \text{ for } 0 < \frac{y-a}{b} < 1 \\ &= \frac{1}{b}\delta, \text{ for } a < y < a+b. \end{aligned}$$

Thus, $f_Y(y) = \frac{1}{b}$ for $a < y < a+b$. Hence, $Y = U[a, a+b]$.

Replacing b by $b-a$ we see that, if $X = U[0,1]$, then $Y = a + (b-a)X$ is $U[a,b]$.

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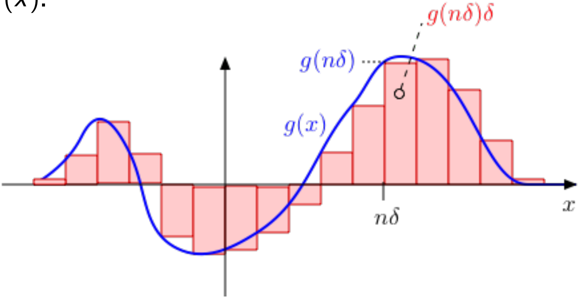

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Extension: $\mathbf{X} = (X_1, \ldots, X_n)$ with $f_{\mathbf{X}}(\mathbf{x})$.

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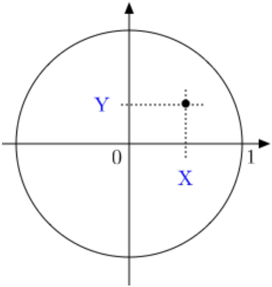

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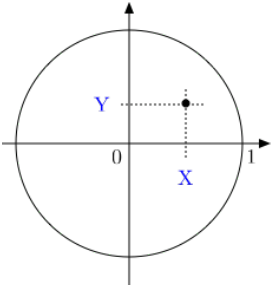

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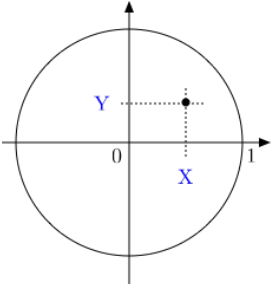

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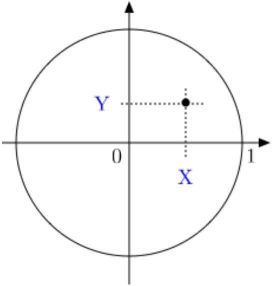

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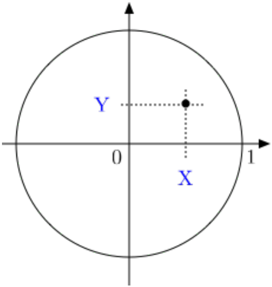

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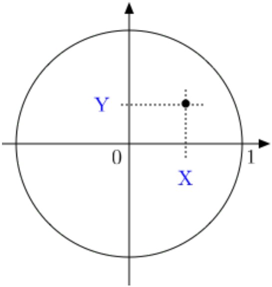

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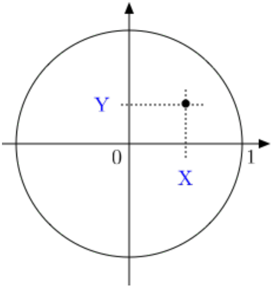

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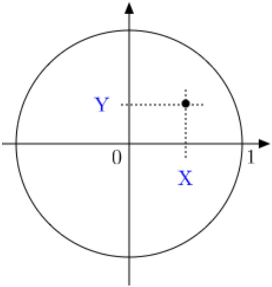

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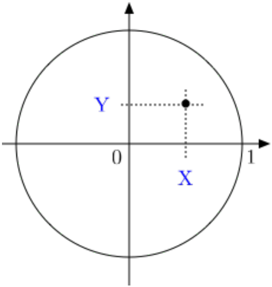

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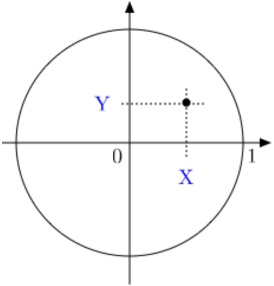

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Since $E[X^0] = 1$, this implies by induction that $E[X^n] = \frac{n!}{\lambda^n}$.

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