CS70: Jean Walrand: Review of Probability.

$\boxed{\text{Probability Review}}$

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Probability Review

1. True or False

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1. True or False
2. Some Key Results

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3. Quiz 1: G

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1. True or False
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3. Quiz 1: G ($\approx$ 40%)

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1. True or False
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6. Common Mistakes

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Probability Review

1. True or False
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3. Quiz 1: G ($\approx$ 40%)
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6. Common Mistakes

True or False

- Ω and A are independent.

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- If $0.3 < \sigma < 3$, then

  $[A_n - 0.6\frac{1}{\sqrt{n}}, A_n + 0.6\frac{1}{\sqrt{n}}] = 95\%$-CI for μ.

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Match Items

[1] $Pr[X \geq a] \leq \frac{E[f(X)]}{f(a)}$

[2] $Pr[|X - E[X]| > a] \leq \frac{var[X]}{a^2}$

[3] $Pr[X \geq a] \leq \min_{\theta > 0} \frac{E[e^{\theta X}]}{e^{\theta a}}$

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- WLLN

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- WLLN (7)

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- WLLN (7)
- MMSE

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- WLLN (7)
- MMSE (6)
- Projection property

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- WLLN (7)
- MMSE (6)
- Projection property (8)
- Chebyshev

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- Chebyshev (2)
- LLSE

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- WLLN (7)
- MMSE (6)
- Projection property (8)
- Chebyshev (2)
- LLSE (5)
- Markov's inequality

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- MMSE (6)
- Projection property (8)
- Chebyshev (2)
- LLSE (5)
- Markov's inequality (1)

Quiz 1: G

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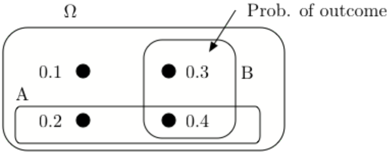

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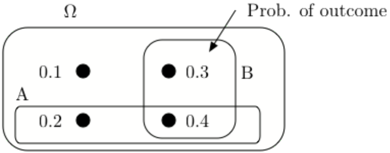

1. What is $P[A|B]$?

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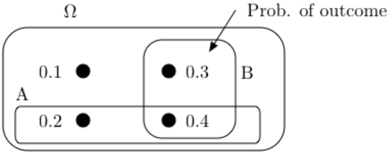

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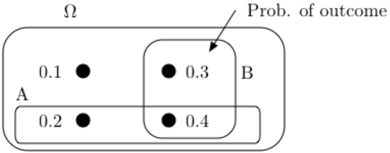
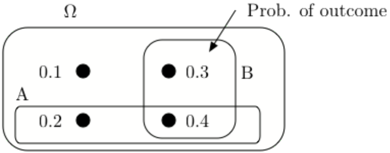


1. What is $P[A|B]$?

   $Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} =$

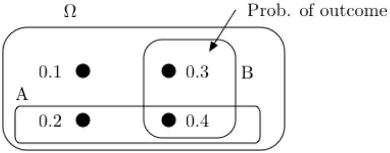
Quiz 1: G



1. What is $P[A|B]$?

$$Pr[A|B] = \frac{Pr[A \cap B]}{Pr[B]} = \frac{0.4}{0.7}$$

Quiz 1: G



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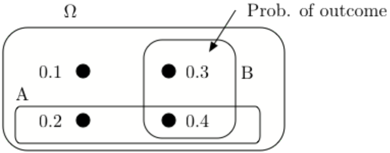
2. What is $Pr[B|A]$?

Quiz 1: G



1. What is $P[A|B]$?

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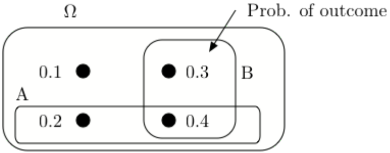
Quiz 1: G



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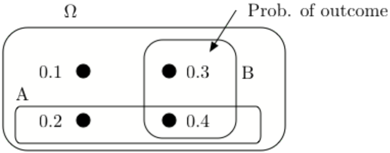
Quiz 1: G



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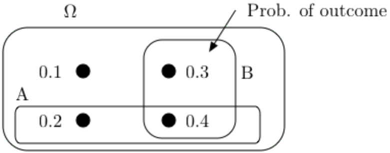
Quiz 1: G



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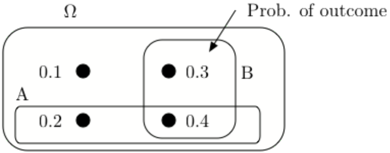
3. Are A and B positively correlated?

Quiz 1: G



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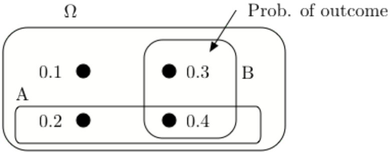
   No.

Quiz 1: G



1. What is $P[A|B]$?

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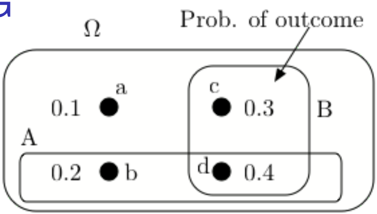
   $Pr[B|A] = \frac{Pr[A \cap B]}{Pr[A]} = \frac{0.4}{0.6}$

3. Are A and B positively correlated?

   No. $Pr[A \cap B] = 0.4 < Pr[A]Pr[B] = 0.6 \times 0.7$.

Quiz 1: G

Quiz 1: G



ω	$X(\omega)$	$Y(\omega)$
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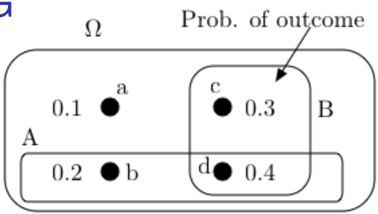
Quiz 1: G



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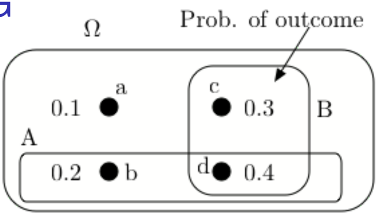
4. What is $E[Y|X]$?

Quiz 1: G



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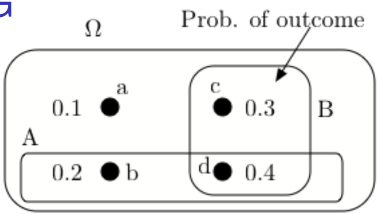
$$E[Y|X=0] \quad =$$

Quiz 1: G



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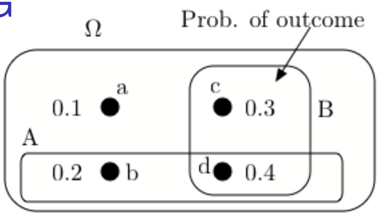
$$E[Y|X=0] = 0 \times Pr[Y=0|X=0] + 2 \times Pr[Y=2|X=0]$$

Quiz 1: G



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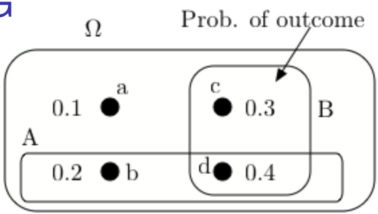
Quiz 1: G



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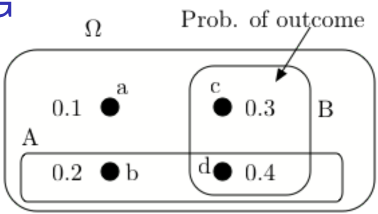
Quiz 1: G



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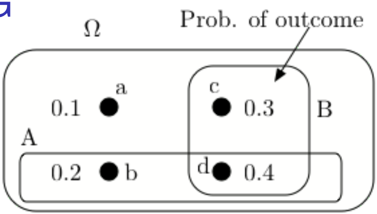
Quiz 1: G



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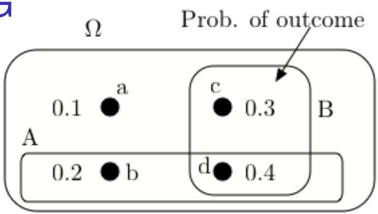
Quiz 1: G



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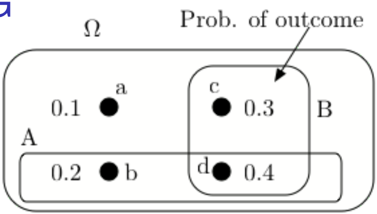

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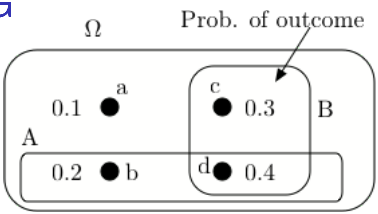
Quiz 1: G



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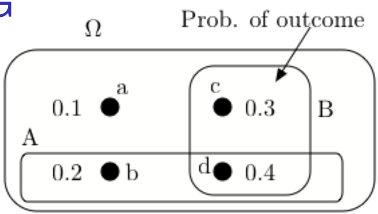
Quiz 1: G



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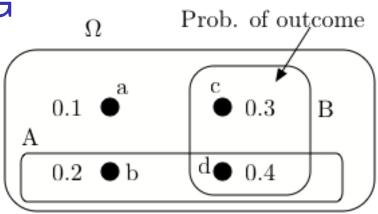
Quiz 1: G



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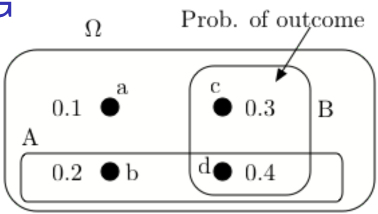
5. What is $cov(X,Y)$?

Quiz 1: G



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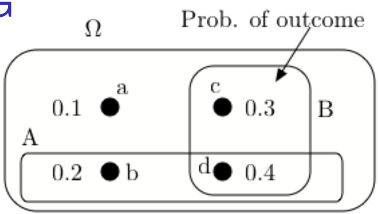
5. What is $cov(X,Y)$?
   $cov(X,Y) =$

Quiz 1: G



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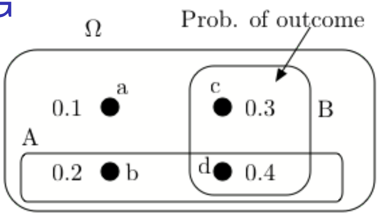
5. What is $cov(X,Y)$?

$cov(X,Y) = E[XY] - E[X]E[Y] =$

Quiz 1: G



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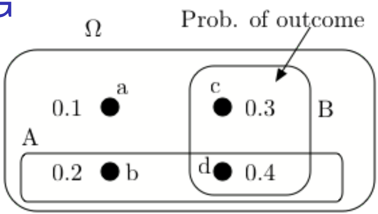
5. What is $cov(X, Y)$?

$cov(X, Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 =$

Quiz 1: G



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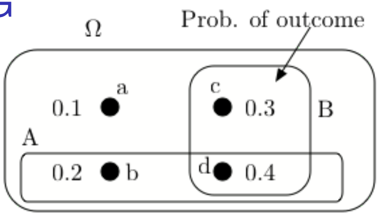
$cov(X,Y) = E[XY] - E[X]E[Y] = 0.8 - 0.6 \times 1.4 = -0.04$

Quiz 1: G



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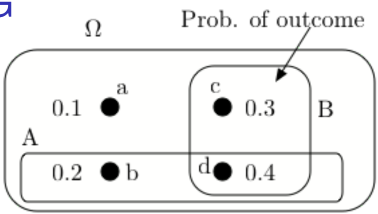
   $cov(X,Y)=E[XY]-E[X]E[Y]=0.8-0.6\times 1.4=-0.04$

6. What is $L[Y|X]$?

Quiz 1: G



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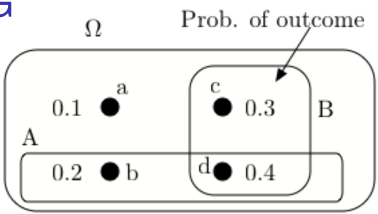
Quiz 1: G



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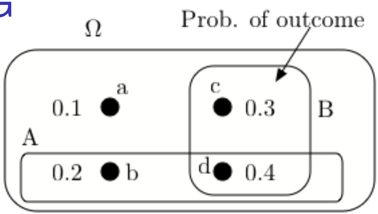
   $L[Y|X] = E[Y] + \frac{cov(X,Y)}{var(X)}(X - E[X]) =$

Quiz 1: G



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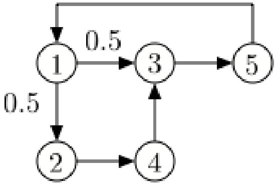
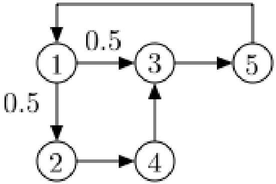
   $cov(X,Y)=E[XY]-E[X]E[Y]=0.8-0.6\times 1.4=-0.04$
6. What is $L[Y|X]$?
   $L[Y|X]=E[Y]+\frac{cov(X,Y)}{var(X)}(X-E[X])=1.4+\frac{-0.04}{0.6\times 0.4}(X-0.6)$

Quiz 1: G

Quiz 1: G

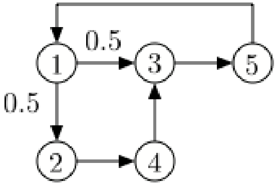


Quiz 1: G



7. Is this Markov chains irreducible?

Quiz 1: G



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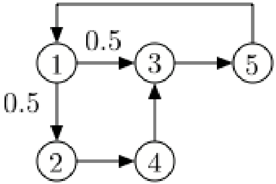
Quiz 1: G



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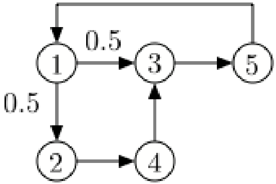
Quiz 1: G



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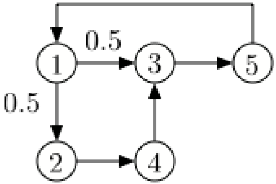
   No.

Quiz 1: G



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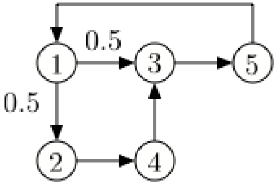
Quiz 1: G



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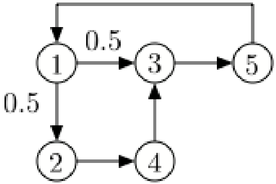
Quiz 1: G



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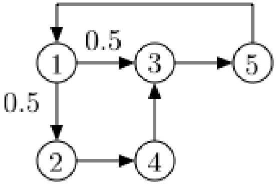

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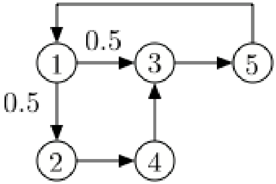
Quiz 1: G



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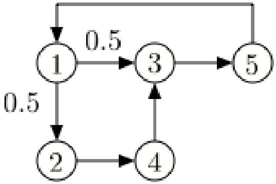

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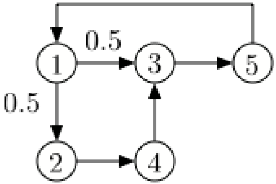
Quiz 1: G



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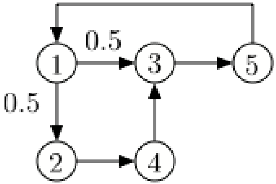

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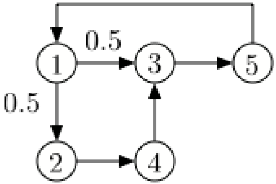
Quiz 1: G



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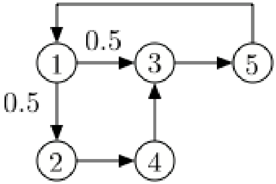
    Let $a = \pi(1)$.

Quiz 1: G



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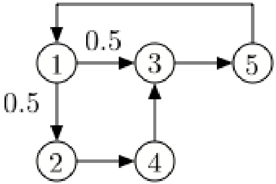
11. Calculate π.

    Let $a = \pi(1)$. Then $a = \pi(5)$,

Quiz 1: G



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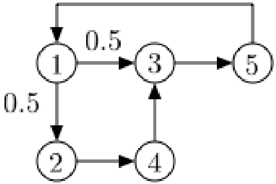
    Let $a = \pi(1)$. Then $a = \pi(5), \pi(2) = 0.5a$,

Quiz 1: G



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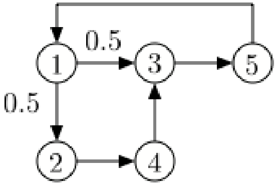
    Let $a = \pi(1)$. Then $a = \pi(5), \pi(2) = 0.5a, \pi(4) = \pi(2) = 0.5a,$

Quiz 1: G



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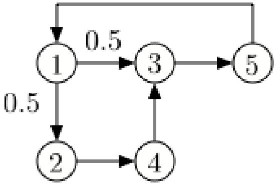
Quiz 1: G



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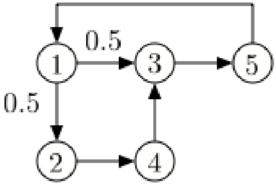
Quiz 1: G



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11. Calculate π.

    Let $a = \pi(1)$. Then $a = \pi(5), \pi(2) = 0.5a, \pi(4) = \pi(2) = 0.5a, \pi(3) = 0.5\pi(1) + \pi(4) = a$. Thus,
    $\pi = [a, 0.5a, a, 0.5a, a] = [1, 0.5, 1, 0.5, 1]a$, so $a = 1/4$.

Quiz 1: G

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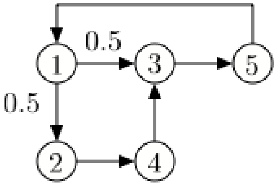

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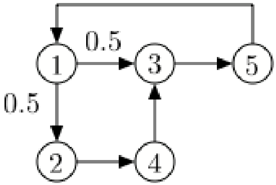
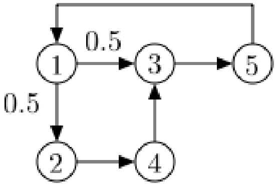


12. Write the first step equations for calculating the mean time from 1 to 4.

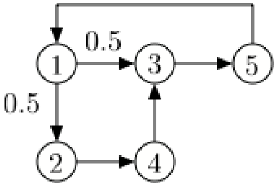
Quiz 1: G



12. Write the first step equations for calculating the mean time from 1 to 4.

$$\beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3)$$

Quiz 1: G



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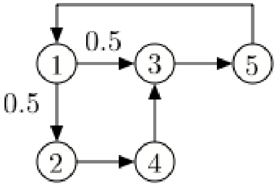
$$\beta(1) = 1 + 0.5\beta(2) + 0.5\beta(3)$$
$$\beta(2) = 1$$

Quiz 1: G



12. Write the first step equations for calculating the mean time from 1 to 4.

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$$\beta(3) = 1 + \beta(5)$$

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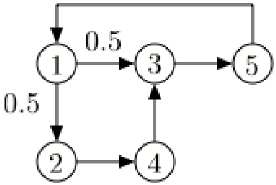

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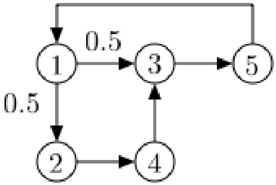

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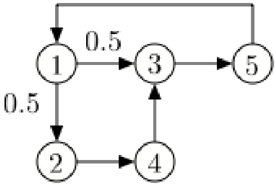

12. Write the first step equations for calculating the mean time from 1 to 4.

$$\begin{aligned}\beta(1) &= 1 + 0.5\beta(2) + 0.5\beta(3)\\ \beta(2) &= 1\\ \beta(3) &= 1 + \beta(5)\\ \beta(5) &= 1 + \beta(1).\end{aligned}$$

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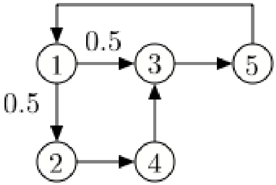
$$\beta(1) \quad = \quad 1 + 0.5 \times 1 + 0.5 \times (1 + (1 + \beta(1)))$$

Quiz 1: G



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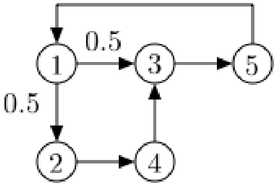

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Hence, $\beta(1) = 5$.

Quiz 1: G

14. Which is $E[Y|X]$? Blue, red or green?

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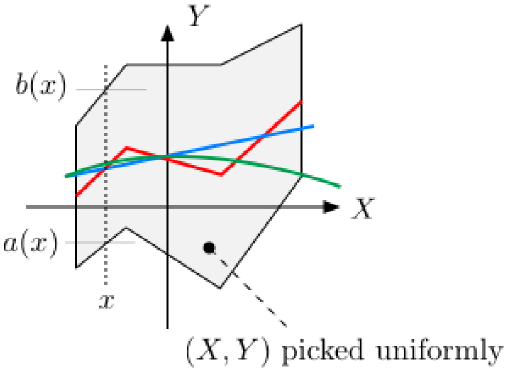

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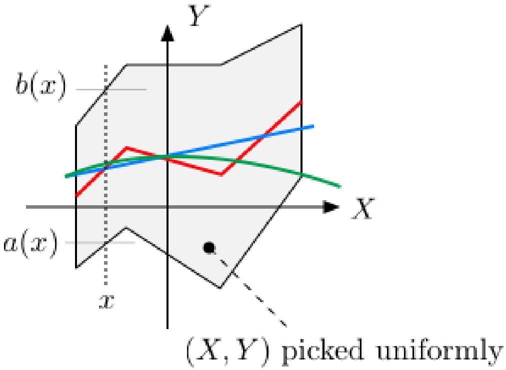

Answer: Red.

Quiz 1: G

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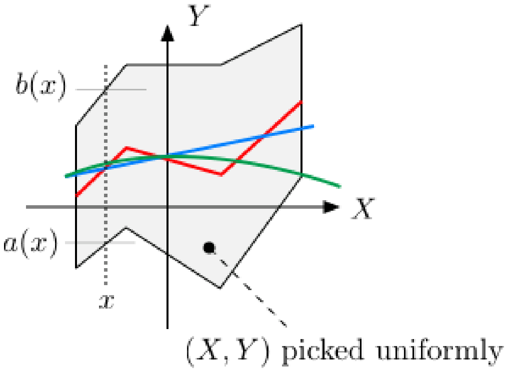

Answer: Red.
Given $X = x, Y = U[a(x), b(x)]$.

Quiz 1: G

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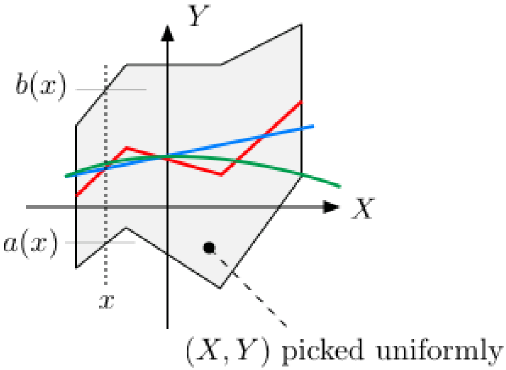
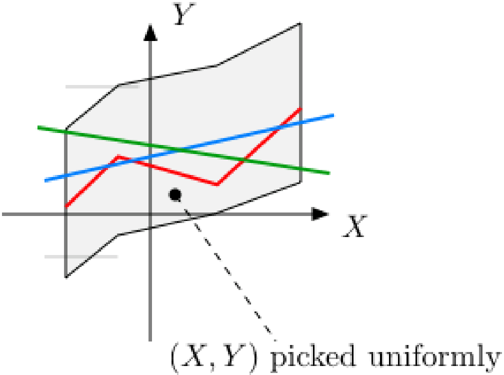


Answer: Red.
Given $X = x$, $Y = U[a(x), b(x)]$. Thus, $E[Y|X = x] = \frac{a(x)+b(x)}{2}$.

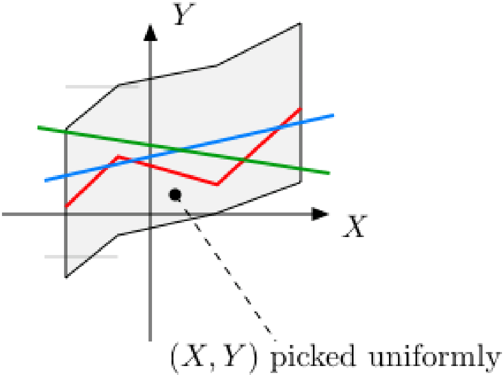
Quiz 1: G

15. Which is $L[Y|X]$? Blue, red or green?



Quiz 1: G

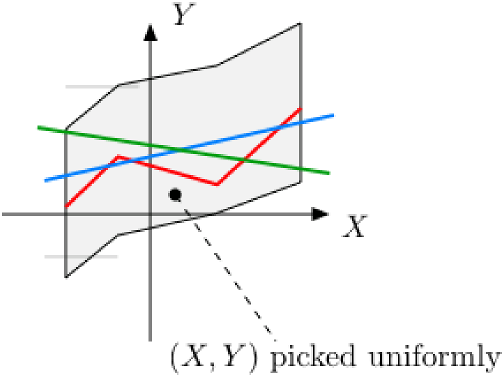
15. Which is $L[Y|X]$? Blue, red or green?



Answer: Blue.

Quiz 1: G

15. Which is $L[Y|X]$? Blue, red or green?



Answer: Blue.
Cannot be red (not a straight line).

Quiz 1: G

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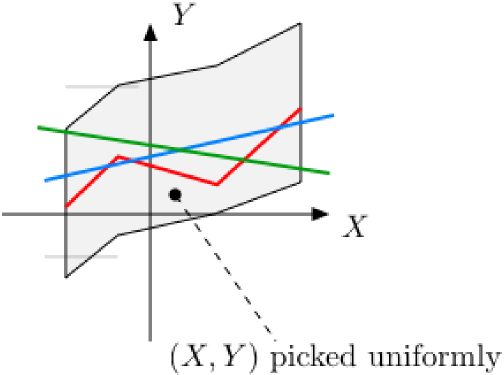

Answer: Blue.
Cannot be red (not a straight line).
Cannot be green: X and Y are clearly positively correlated.

Quiz 2: PG

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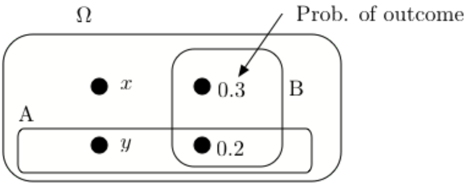
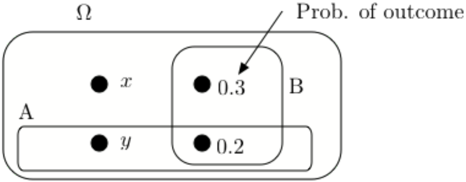
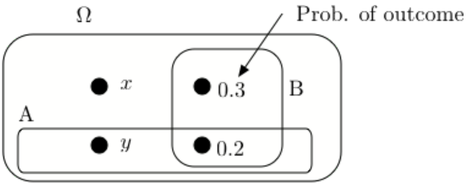


Quiz 2: PG



1. Find (x, y) so that A and B are independent.

Quiz 2: PG



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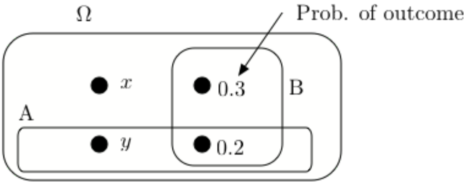
   We need
   $$Pr[A \cap B] = Pr[A]Pr[B]$$

Quiz 2: PG



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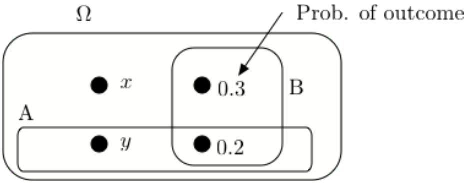
$$0.2 = (y + 0.2) \times 0.5 =$$

Quiz 2: PG



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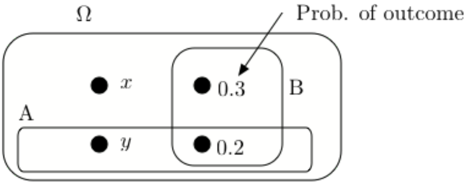

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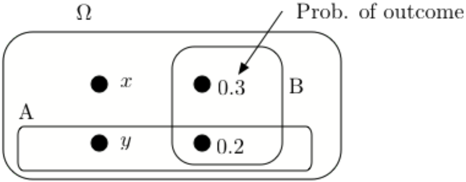
   Hence,
   $$y = 0.2$$

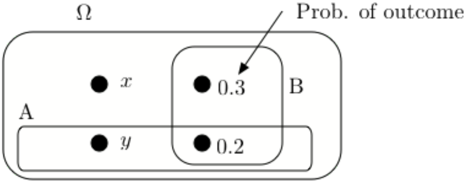
Quiz 2: PG



1. Find (x, y) so that A and B are independent.

   We need
   $$Pr[A \cap B] = Pr[A]Pr[B]$$
   That is,
   $$0.2 = (y + 0.2) \times 0.5 = 0.5y + 0.1$$
   Hence,
   $$y = 0.2 \text{ and } x =$$

Quiz 2: PG



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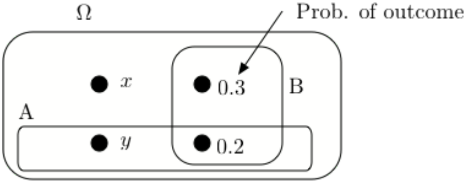

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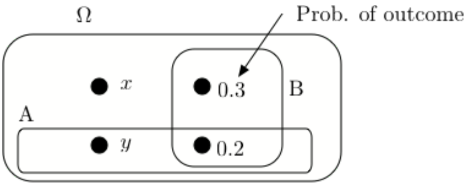

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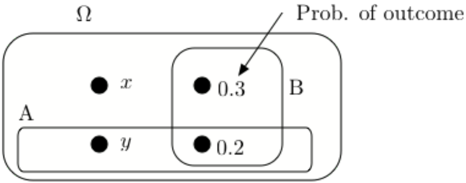

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Quiz 2: PG

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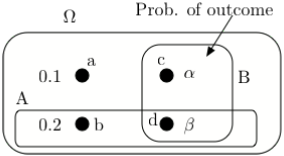
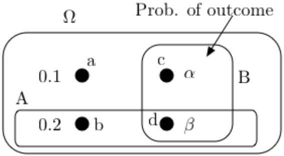


ω	$X(\omega)$	$Y(\omega)$
a	0	0
b	1	0
c	0	2
d	1	2

Quiz 2: PG



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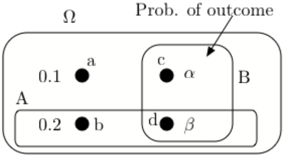
Quiz 2: PG



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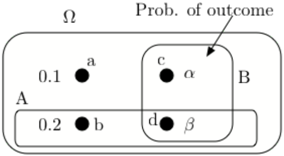

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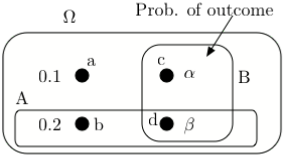
Quiz 2: PG



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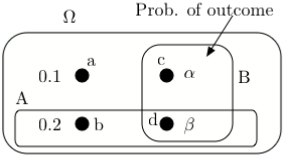

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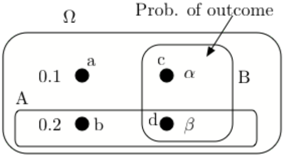
Quiz 2: PG



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Quiz 2: PG

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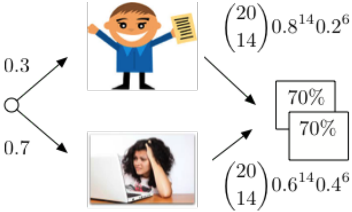

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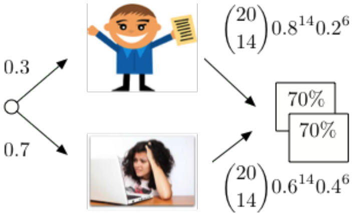

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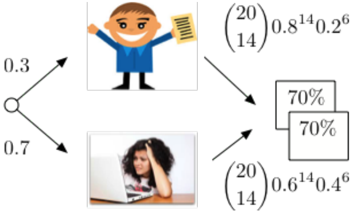

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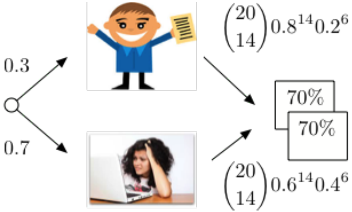

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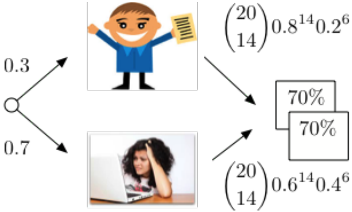

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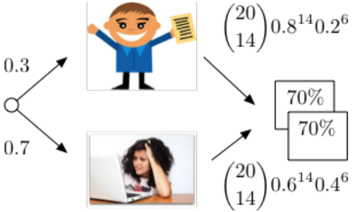

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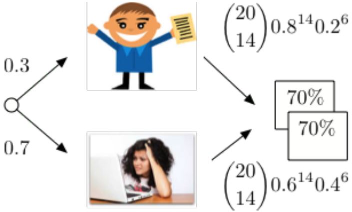

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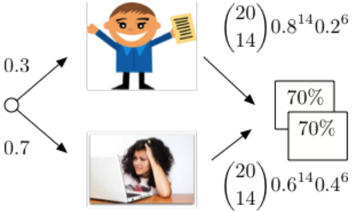

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Quiz 2: PG

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Then
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Quiz 2: PG

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Hence,

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Quiz 2: PG

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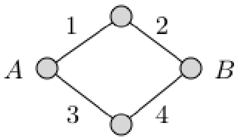

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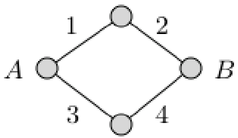

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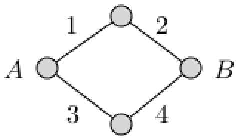

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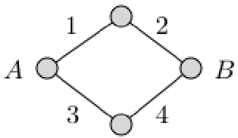

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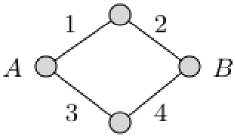

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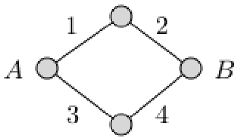

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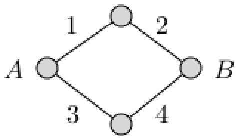

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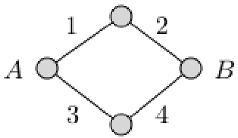
Quiz 3: R



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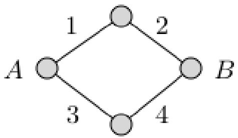

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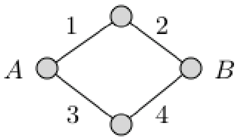

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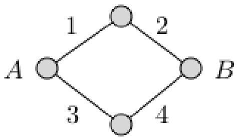

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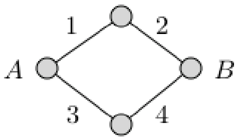

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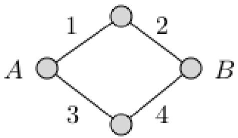

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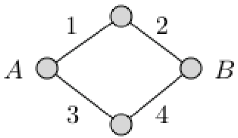
Quiz 3: R



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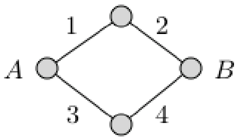

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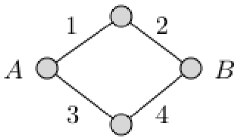
Quiz 3: R



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\end{aligned}
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Quiz 3: R

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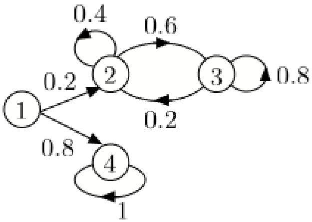

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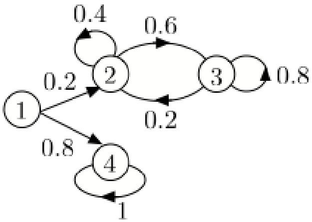

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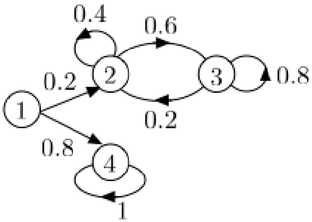
Quiz 3: R



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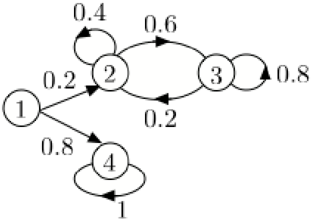

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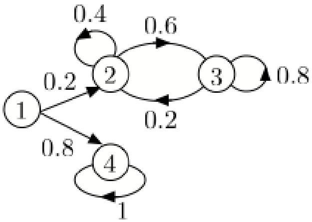
Quiz 3: R



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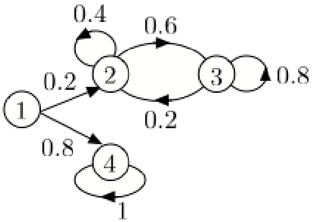

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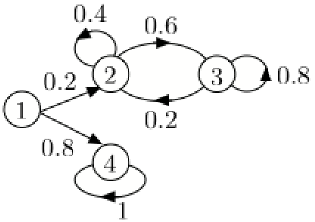
Quiz 3: R



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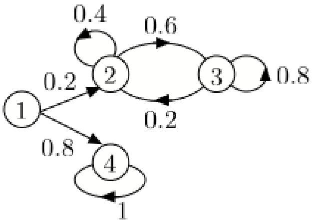
$$\frac{0.2}{0.2+0.6} = 0.25.$$

Quiz 3: R



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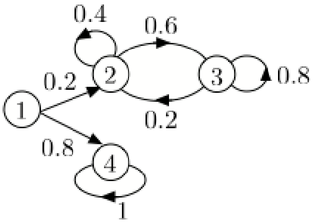

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$$[0, 0.25\alpha, 0.75\alpha, 1 - \alpha].$$

Quiz 3: R

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4. A bag has n red and n blue balls. You pick two balls (no replacement). Let $X = 1$ if ball 1 is red and $X = -1$ otherwise. Define Y likewise for ball 2.
   $\rightarrow$ Are X and Y positively, negatively, or un- correlated?

   Clearly, negatively.

5. Calculate $cov(X,Y)$.

   $cov(X,Y) = E[XY] - E[X]E[Y]$

   $E[X] = E[Y]$, by symmetry

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Thus,

$$E[Y|X] = (2\alpha - 1)X = -\frac{1}{2n-1}X.$$

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