Homework Logistics

- HW1 is due this Friday 09/02.
- Login to Gradescope *TODAY* to see if you have access to CS 70. If you still do not have access, read https://piazza.com/class/irwxcmgdofp2uz?cid=6.
- Homework must be submitted electronically to Gradescope as pdf but it may be prepared by hand, in LaTeX, or using Microsoft Word. Make sure the uploaded PDF is readable!!!

Lecture Outline

Strengthening Induction Hypothesis.

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Strengthening Induction Hypothesis.

Strong Induction

Lecture Outline

Strengthening Induction Hypothesis.

Strong Induction

Well ordered principle.

Strengthening Induction Hypothesis.

Theorem: The sum of the first *n* odd numbers is a perfect square.

Strengthening Induction Hypothesis.

Theorem: The sum of the first n odd numbers is a perfect square.

Theorem: The sum of the first n odd numbers is k^2.

Strengthening Induction Hypothesis.

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kth odd number is $2k-1$ for $k \geq 1$.

Strengthening Induction Hypothesis.

Theorem: The sum of the first n odd numbers is a perfect square.

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Base Case 1 (1st odd number) is 1^2.

Strengthening Induction Hypothesis.

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Base Case	1 (1st odd number) is 1^2.
Induction Hypothesis	Sum of first k odds is perfect square $a^2 = k^2$.

Strengthening Induction Hypothesis.

Theorem: The sum of the first n odd numbers is a perfect square.

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kth odd number is $2k-1$ for $k \geq 1$.

Base Case: 1 (1st odd number) is 1^2.

Induction Hypothesis: Sum of first k odds is perfect square $a^2 = k^2$.

Induction Step: To prove that sum of first $k+1$ odds is $(k+1)^2$.

Strengthening Induction Hypothesis.

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1. The $(k+1)$st odd number is $2(k+1)-1 = 2k+1$.

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1. The $(k+1)$st odd number is $2(k+1)-1 = 2k+1$.
2. Sum of the first $k+1$ odds is $a^2 + 2k + 1 = k^2 + 2k + 1$

Strengthening Induction Hypothesis.

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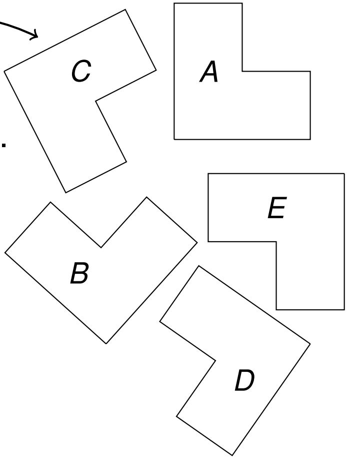
1. The $(k+1)$st odd number is $2(k+1)-1 = 2k+1$.
2. Sum of the first $k+1$ odds is $a^2+2k+1 = k^2+2k+1$
3. $k^2+2k+1 = (k+1)^2$
   ... P(k+1)!

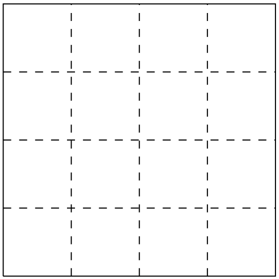
□

Tiling Cory Hall Courtyard.

Use these L-tiles.



To Tile this 4×4 courtyard.



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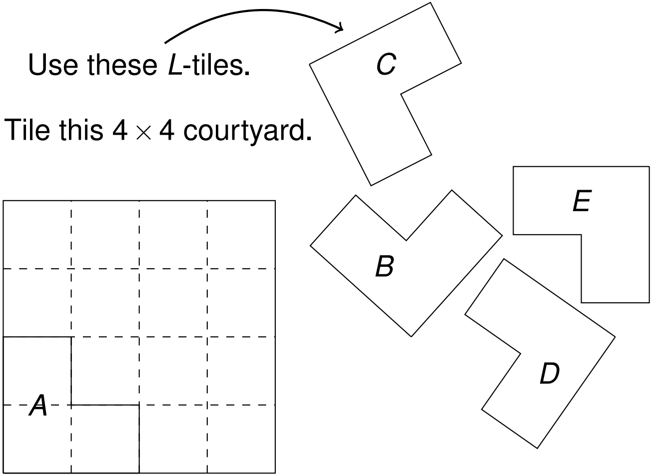

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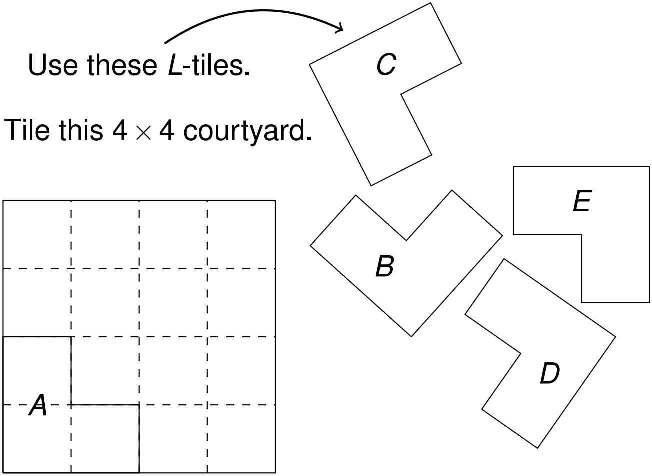

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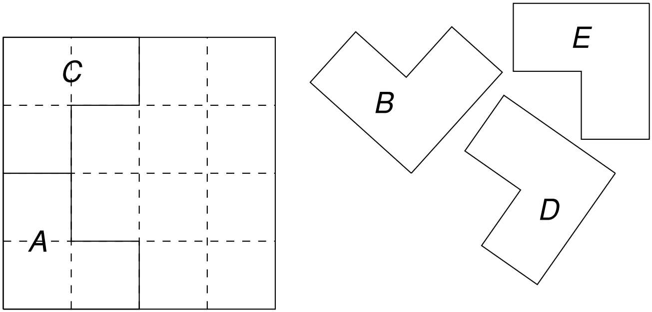

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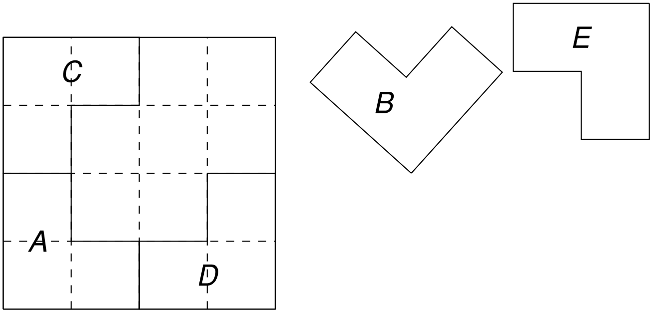

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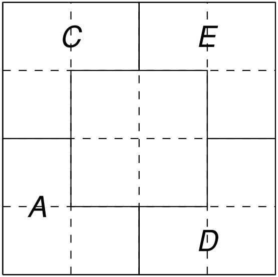

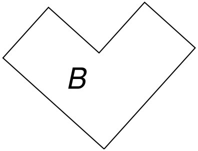

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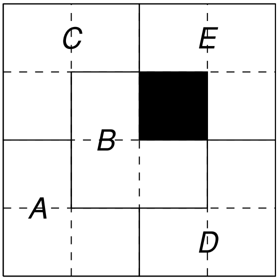

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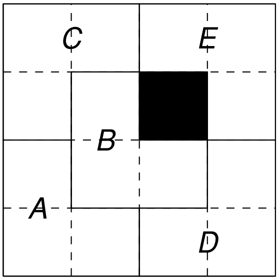

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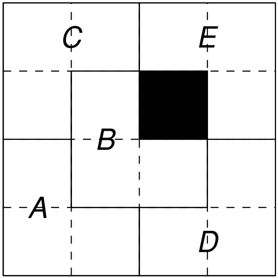
Use these L-tiles.

To Tile this 4×4 courtyard.



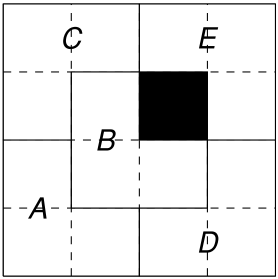
Alright!

Tiled 4×4 square with 2×2 L-tiles.

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.



Alright!

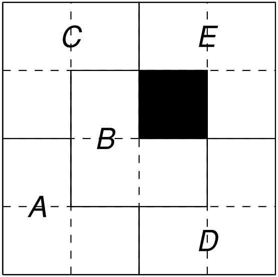
Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Tiling Cory Hall Courtyard.

Use these L-tiles.

To Tile this 4×4 courtyard.



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with a center hole.

Can we tile any $2^n \times 2^n$ with L-tiles (with a hole)

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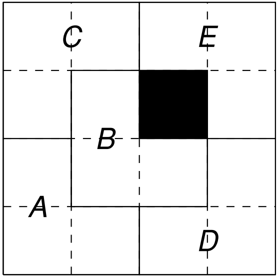

Alright!

Tiled 4×4 square with 2×2 L-tiles.

with a center hole.

Can we tile any $2^n \times 2^n$ with L-tiles (with a hole) for every n!

Hole have to be there? Maybe just one?

Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

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Theorem: Any tiling of $2^n \times 2^n$ square has to have one hole.

Proof: Each tile covers 3 squares. The remainder of 2^{2n} divided by 3 is 1.

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Base case: true for $n = 0$.

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Ind Hyp: $n = k$. $2^{2k} = 3a + 1$ for integer a.

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$$2^{2(k+1)}$$

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$$2^{2(k+1)} = 2^{2k} * 2^2$$

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$$\begin{aligned} 2^{2(k+1)} &= 2^{2k} * 2^2 \\ &= 4 * 2^{2k} \end{aligned}$$

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&= 4 * (3a + 1)
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a integer

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Hole in center?

Theorem: Can tile the $2^n \times 2^n$ square to leave a hole adjacent to the center.

Proof:

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The hole is adjacent to the center of the 2×2 square.

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Induction Hypothesis:

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Induction Hypothesis:
Any $2^n \times 2^n$ square can be tiled with a hole at the center.

Hole in center?

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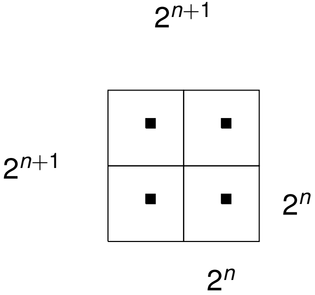

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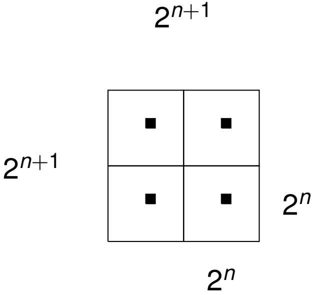

What to do now???

Hole can be anywhere!

Theorem: Can tile the $2^n \times 2^n$ to leave a hole adjacent *anywhere.*

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Better theorem

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Induction Hypothesis:
"Any $2^n \times 2^n$ square can be tiled with a hole **anywhere.**"
Consider $2^{n+1} \times 2^{n+1}$ square.

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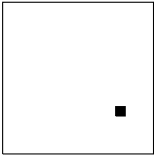

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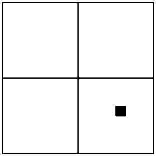

Use induction hypothesis in each.

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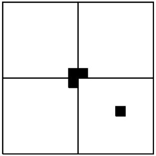

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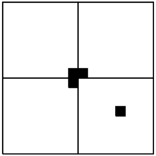

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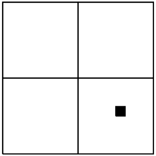

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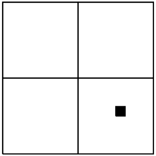

Use induction hypothesis in each.

Use L-tile and ... we are done. $\square$

Strong Induction: Example

Theorem: Every natural number $n > 1$ is either a prime or can be written as a product of primes.

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Either $n+1$ is a prime or $n+1 = a \cdot b$ where $1 < a, b < n+1$.

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Theorem: Every natural number $n > 1$ is either a prime or can be written as a product of primes.

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The **Well ordering principle** states that for any subset of the natural numbers there is a smallest element.

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Slight differences: showed for all $n \geq 16$ that $\wedge_{i=4}^{n-1} P(i) \implies P(n)$.

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How about $P(1) \implies P(2)$?

Horses of the same color...

Theorem: All horses have the same color.

Base Case: $P(1)$ - trivially true.
New Base Case: $P(2)$: there are two horses with same color.

Induction Hypothesis: $P(k)$ - Any k horses have the same color.

Induction step $P(k+1)$?
First k have same color by $P(k)$.
Second k have same color by $P(k)$.
A horse in the middle in common!

Fix base case.

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As we will see, it is more subtle to catch errors in proofs of correct theorems!!

Summary: principle of induction.

Today: More induction.

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$(P(0)$

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$(P(0) \wedge ((\forall k \in N)(P(k) \implies P(k+1))))$

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$(P(0) \wedge ((\forall n \leq k P(n)) \implies P(k+1)))$

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Also Today: strengthened induction hypothesis.

Strengthen theorem statement.
Sum of first n odds is n^2.
Hole anywhere.
Not same as strong induction.

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Induction $\equiv$ Recursion.