Lecture 6: Graphs.

Graphs!

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Euler

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Graphs!
- Euler
- Definitions: model.

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- Definitions: model.
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Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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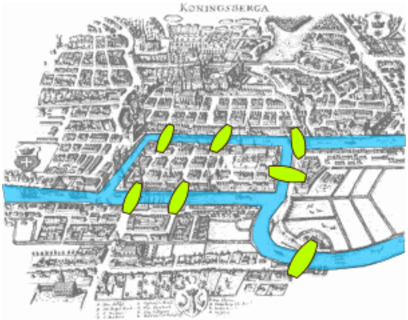

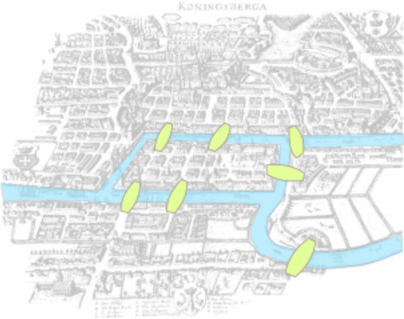

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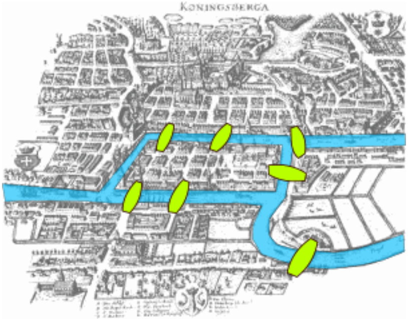

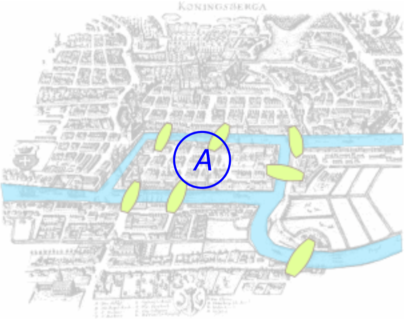

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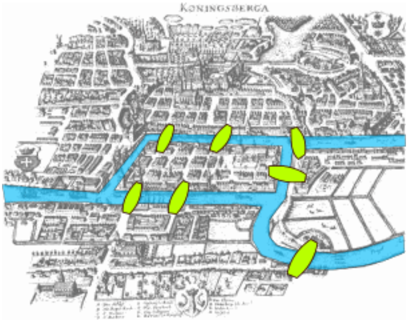

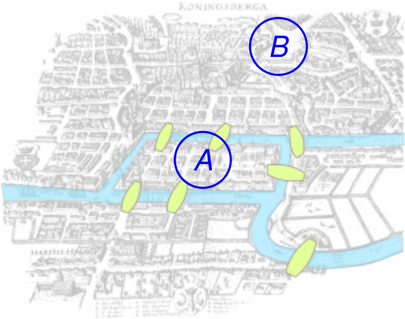

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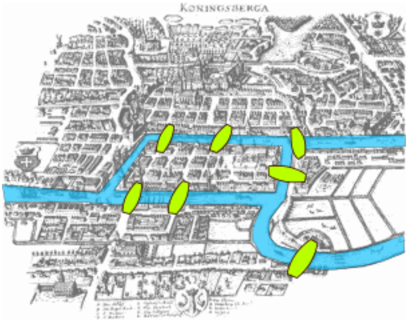

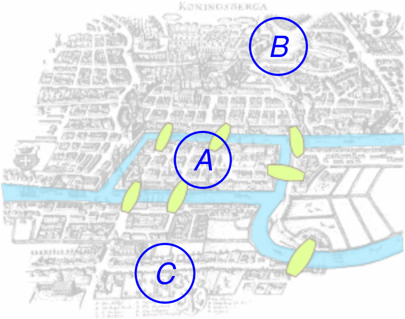

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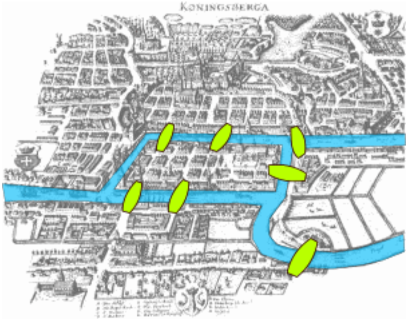

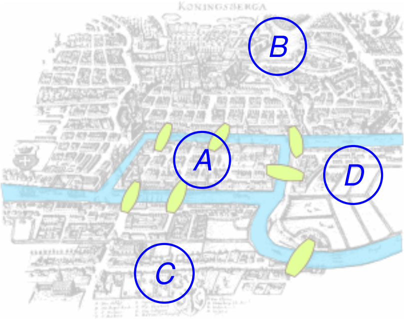

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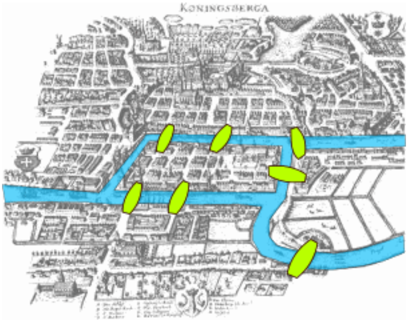

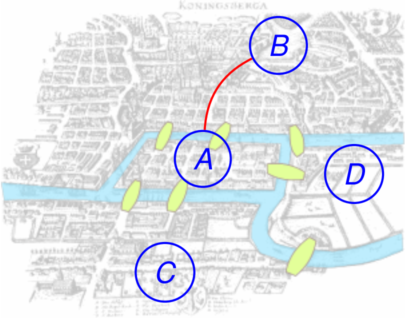

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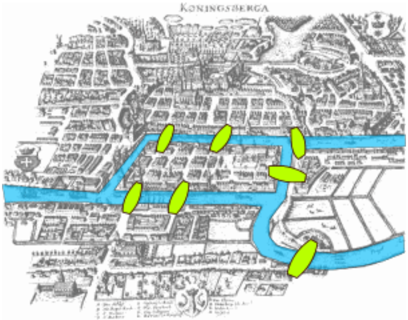

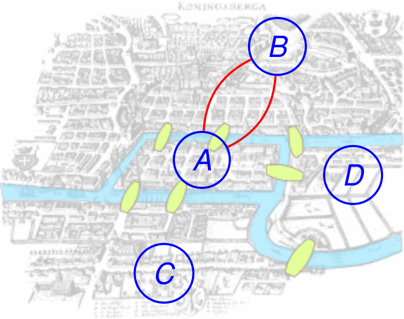

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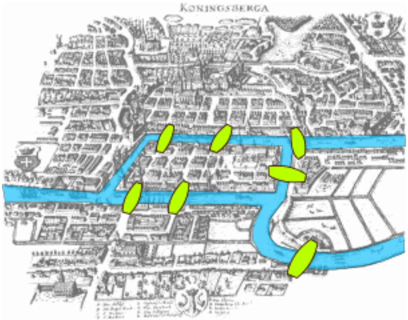

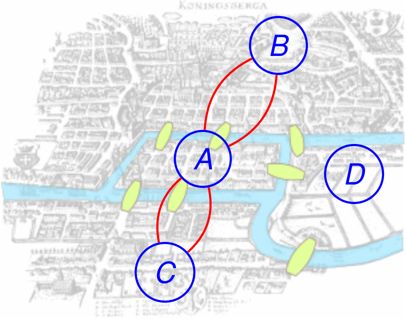

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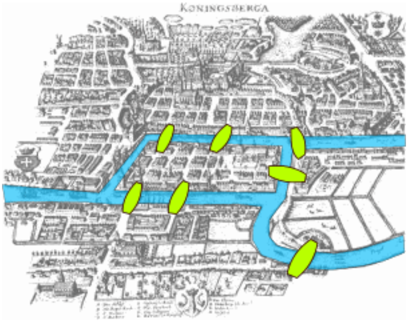

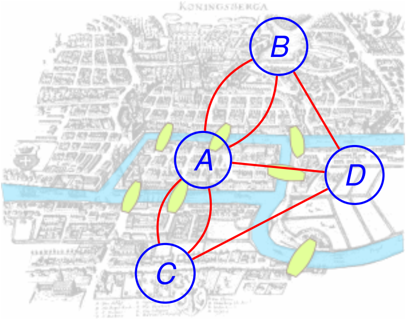

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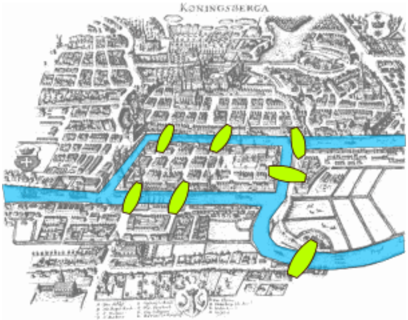

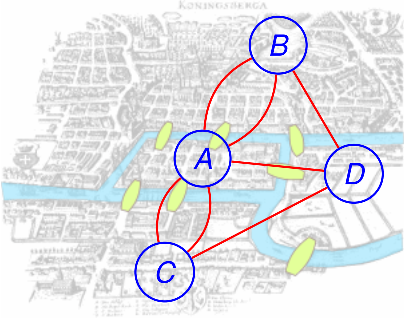

Can you draw a tour in the graph where you visit each edge once?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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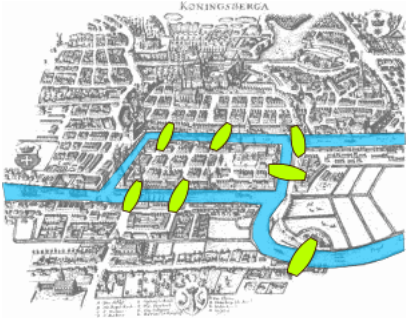

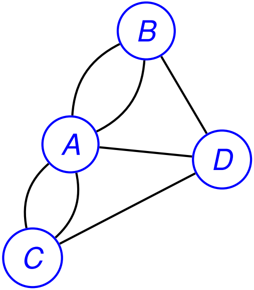

Can you draw a tour in the graph where you visit each edge once? Yes?

Konigsberg bridges problem.

Can you make a tour visiting each bridge exactly once?

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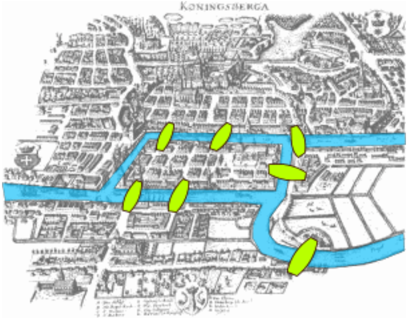
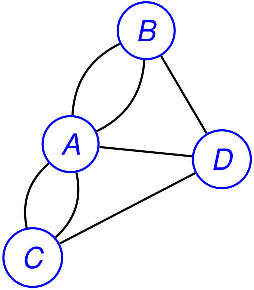
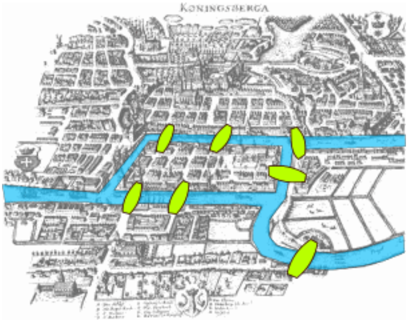


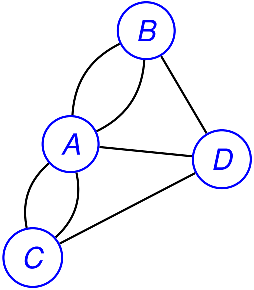


Can you draw a tour in the graph where you visit each edge once? Yes? No?

Konigsberg bridges problem.

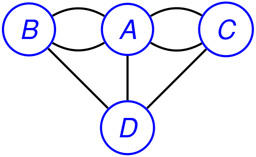
Can you make a tour visiting each bridge exactly once?





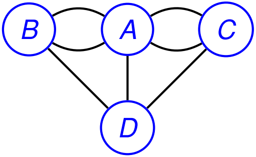
Can you draw a tour in the graph where you visit each edge once?   Yes?   No?
We will see!

Graphs: formally.



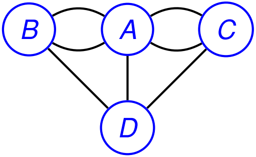
Graph:

Graphs: formally.



Graph: $G = (V, E)$.

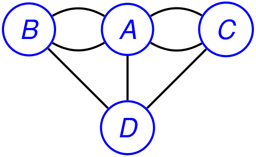
Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

Graphs: formally.

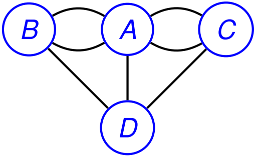


Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

Graphs: formally.



Graph: $G = (V, E)$.

V - set of vertices.

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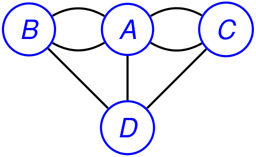
$E \subseteq V \times V$ -

Graphs: formally.



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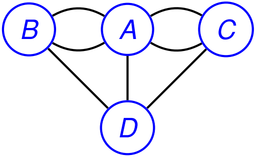

Graph: $G = (V, E)$.

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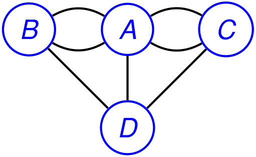
$\{A, B, C, D\}$

$E \subseteq V \times V$ - set of edges.

$\{\{A, B\}$

Graphs: formally.



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$\{\{A, B\}, \{A, B\}$

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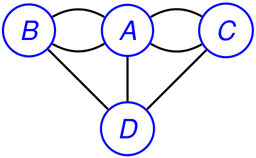

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$\{\{A, B\}, \{A, B\}, \{A, C\},$

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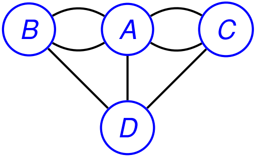

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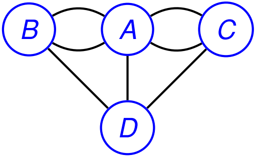

Graph: $G = (V, E)$.

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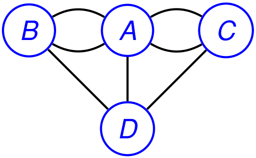
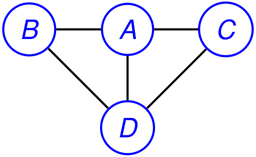
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

$\{A, B, C, D\}$

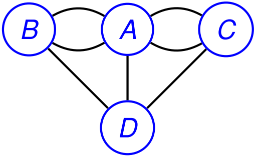
$E \subseteq V \times V$ - set of edges.

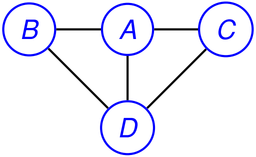
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

Graphs: formally.





Graph: $G = (V, E)$.

V - set of vertices.

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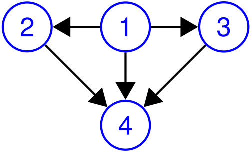
$\{\{A, B\}, \{A, B\}, \{A, C\}, \{A, C\}, \{B, D\}, \{A, D\}, \{C, D\}\}$.

For CS 70, usually simple graphs.

No parallel edges.

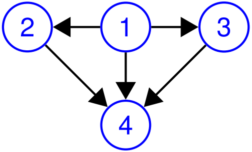
Multigraph above.

Directed Graphs



$G = (V, E)$.

Directed Graphs



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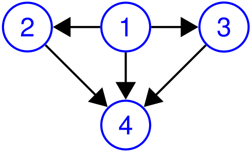
V - set of vertices.

Directed Graphs

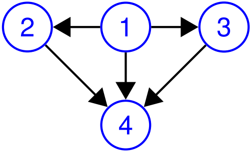


$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

Directed Graphs



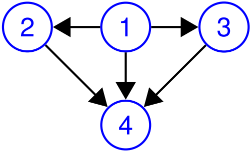
$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

E ordered pairs of vertices.

Directed Graphs



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E ordered pairs of vertices.

$\{(1, 2),$

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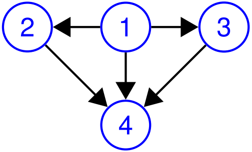

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V - set of vertices.

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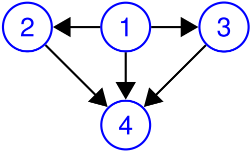
E ordered pairs of vertices.

$\{(1, 2), (1, 3),$

Directed Graphs



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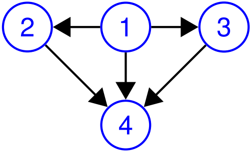
Directed Graphs



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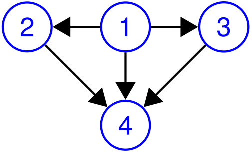
Directed Graphs



$G = (V, E)$.

V - set of vertices.

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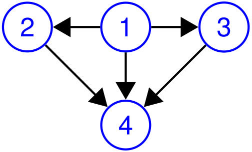
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Directed Graphs



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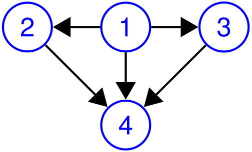
One way streets.
Tournament:

Directed Graphs



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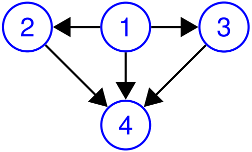
E ordered pairs of vertices.

$\{(1,2),(1,3),(1,4),(2,4),(3,4)\}$

One way streets.

Tournament: 1 beats 2,

Directed Graphs



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V - set of vertices.

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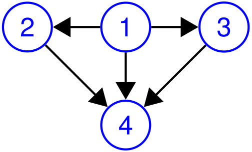
E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence:

Directed Graphs



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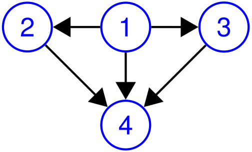
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2,

Directed Graphs



$G = (V, E)$.
V - set of vertices.
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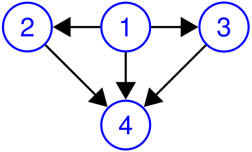
One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Directed Graphs



$G = (V, E)$.

V - set of vertices.

$\{1, 2, 3, 4\}$

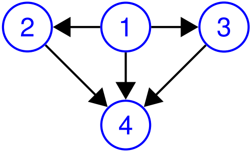
E ordered pairs of vertices.

$\{(1, 2), (1, 3), (1, 4), (2, 4), (3, 4)\}$

One way streets.
Tournament: 1 beats 2, ...
Precedence: 1 is before 2, ..

Social Network:

Directed Graphs



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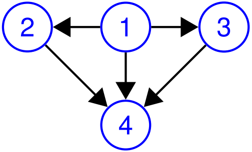
Directed Graphs



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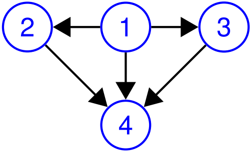

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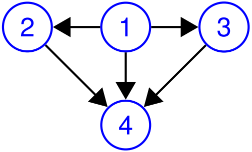
Social Network: Directed? Undirected?
Friends.

Directed Graphs



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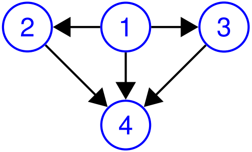

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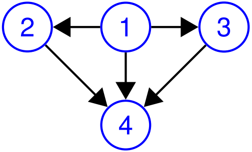
Directed Graphs



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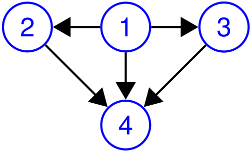

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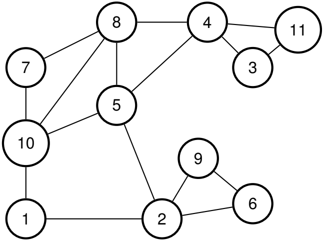

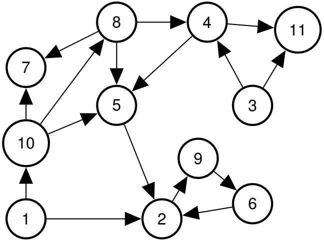

Neighbors of 10?

Graph Concepts and Definitions.

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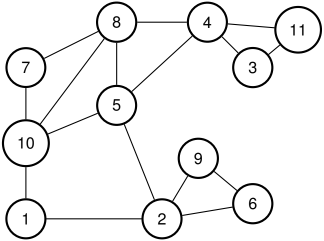

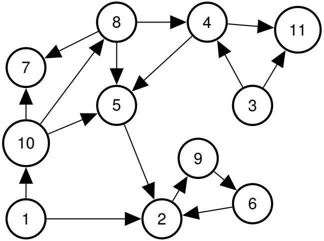

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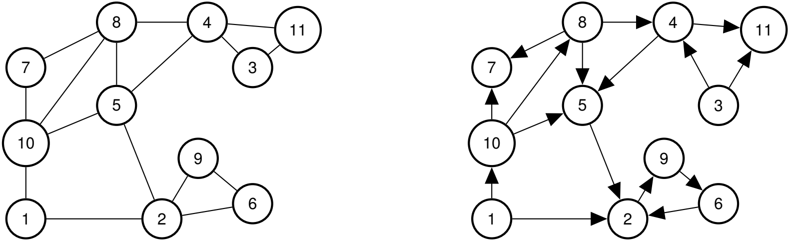

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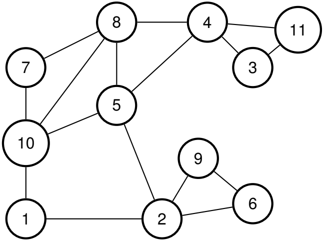

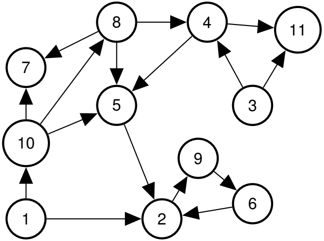

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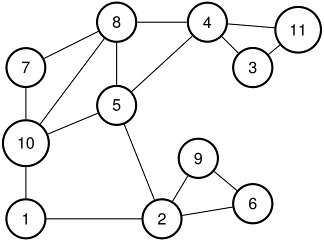

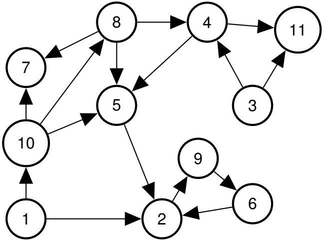

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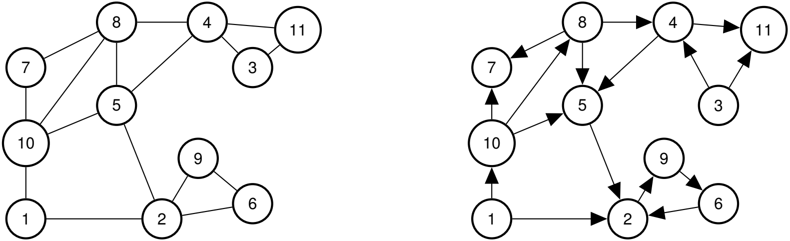

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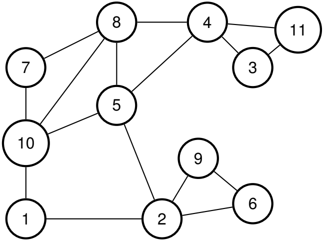

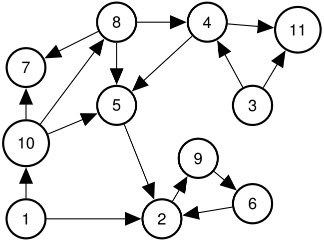

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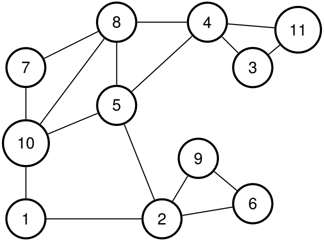

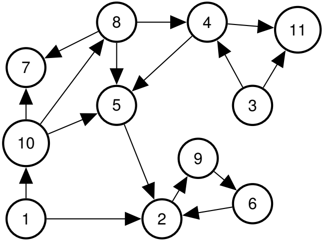

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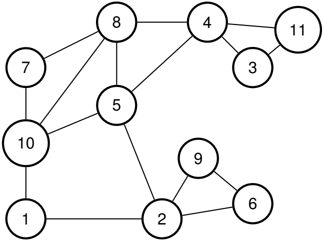

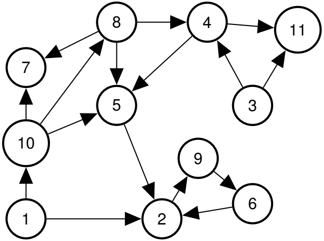

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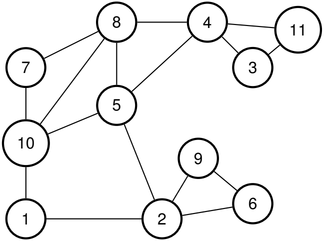

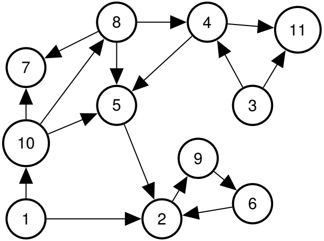

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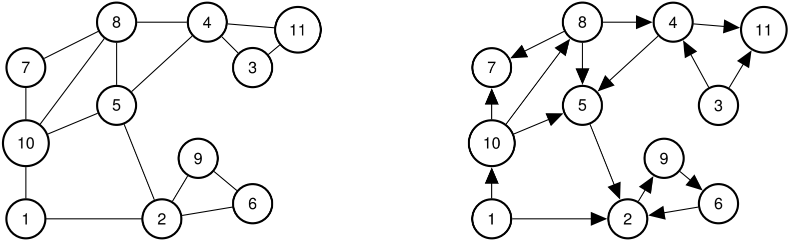

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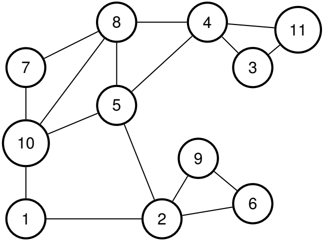

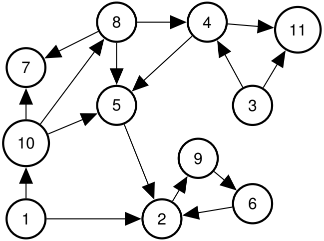

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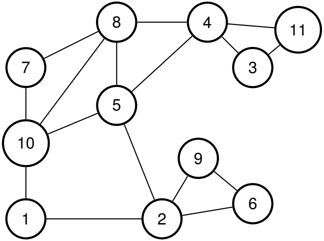

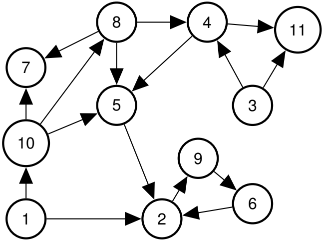

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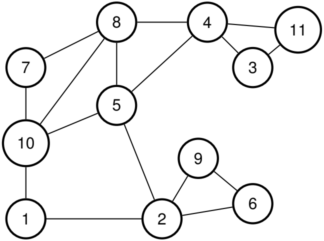

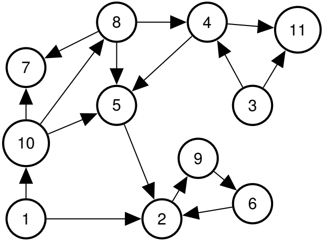

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In-degree of 10?

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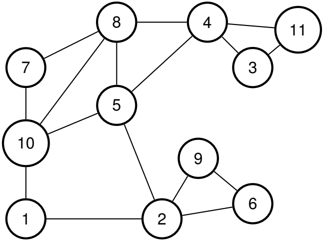

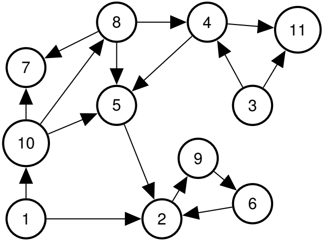

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In-degree of 10? 1

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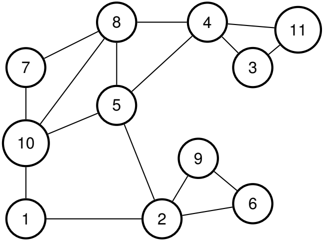

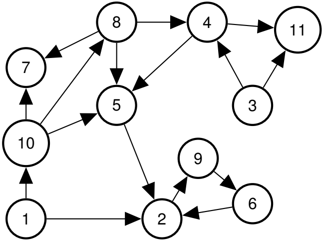

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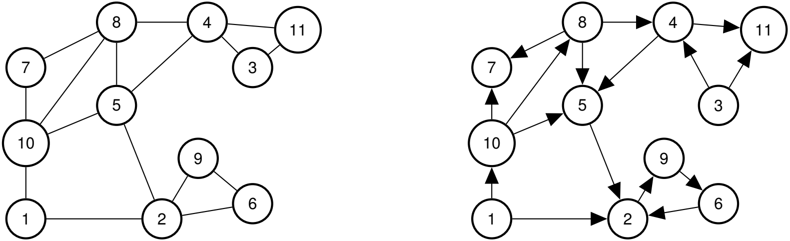

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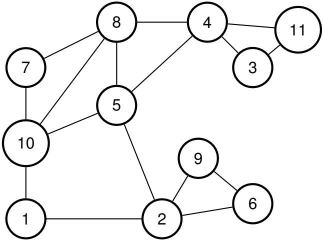

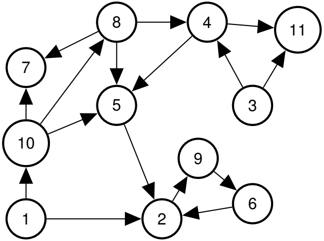

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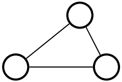

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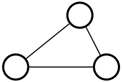

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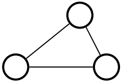

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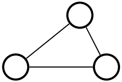


Not (B)! Triangle.

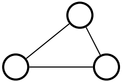
What?

Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



Not (B)! Triangle.

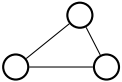
What? For triangle number of edges is 3, the sum of degrees is 6.

Quick Proof.

The sum of the vertex degrees is equal to

(A) the total number of vertices, $|V|$.
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



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What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...

Quick Proof.

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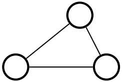

Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$?

Quick Proof.

The sum of the vertex degrees is equal to

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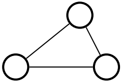
(B) the total number of edges, $|E|$.
(C) What?

Not (A)! Triangle.



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Could it always be...$2|E|$?

How many incidences does each edge contribute?

Quick Proof.

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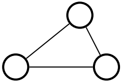

Not (B)! Triangle.

What? For triangle number of edges is 3, the sum of degrees is 6.

Could it always be...$2|E|$?

How many incidences does each edge contribute? 2.

Quick Proof.

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(B) the total number of edges, $|E|$.
(C) What?

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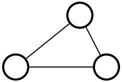

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$2|E|$ incidences are contributed in total!

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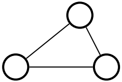

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How many incidences does each edge contribute? 2.
$2|E|$ incidences are contributed in total!
What is degree v?

Quick Proof.

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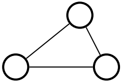 Not (B)! Triangle.

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How many incidences does each edge contribute? 2.
$2|E|$ incidences are contributed in total!
What is degree v? incidences contributed to v!

Quick Proof.

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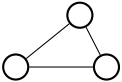

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sum of degrees is total incidences

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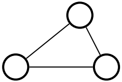

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sum of degrees is total incidences ... or $2|E|$.

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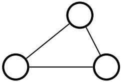 Not (B)! Triangle.

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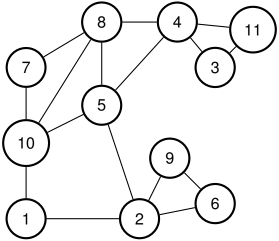
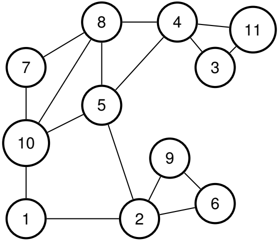
Thm: Sum of vertex degress is $2|E|$.

Paths, walks, cycles, tour.



A path in a graph is a sequence of edges.

Paths, walks, cycles, tour.



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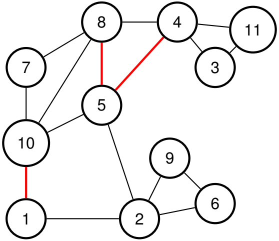
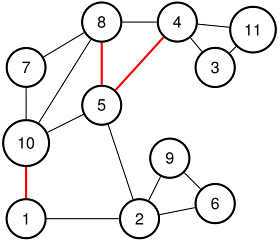


A path in a graph is a sequence of edges.

Path? $\{1, 10\}$, $\{8, 5\}$, $\{4, 5\}$?

Paths, walks, cycles, tour.



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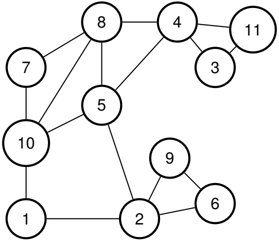

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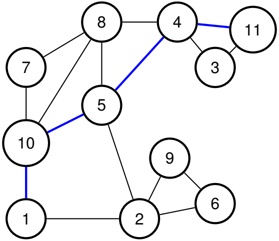
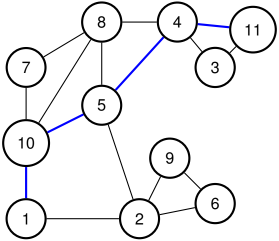


A path in a graph is a sequence of edges.

Path? $\{1, 10\}$, $\{8, 5\}$, $\{4, 5\}$? No!
Path? $\{1, 10\}$, $\{10, 5\}$, $\{5, 4\}$, $\{4, 11\}$?

Paths, walks, cycles, tour.



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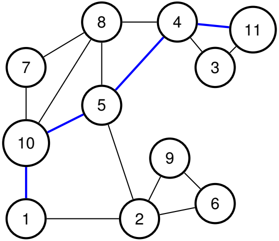

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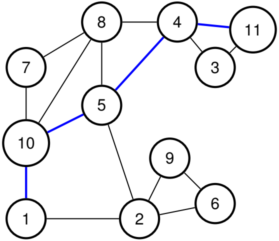
Path: $(v_1,v_2),(v_2,v_3),\ldots(v_{k-1},v_k)$.

Paths, walks, cycles, tour.



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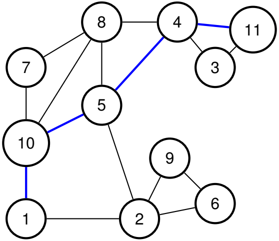
Quick Check!

Paths, walks, cycles, tour.



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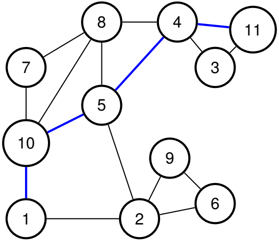
Quick Check! Length of path?

Paths, walks, cycles, tour.



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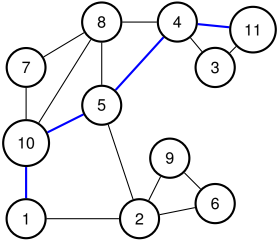
Quick Check! Length of path? k vertices

Paths, walks, cycles, tour.



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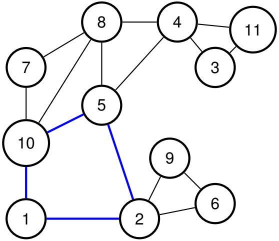
Quick Check! Length of path? k vertices or $k-1$ edges.

Paths, walks, cycles, tour.



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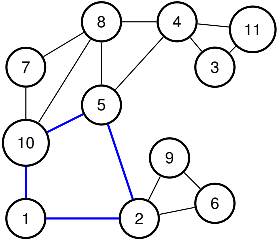
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$.

Paths, walks, cycles, tour.



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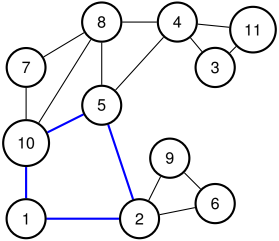
Cycle: Path with $v_1 = v_k$. Length of cycle?

Paths, walks, cycles, tour.



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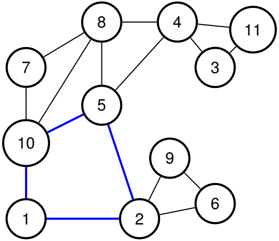
Quick Check! Length of path? k vertices or $k-1$ edges.
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!

Paths, walks, cycles, tour.



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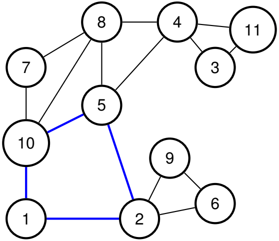
Quick Check! Length of path? k vertices or $k-1$ edges.

Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually *simple*.

Paths, walks, cycles, tour.



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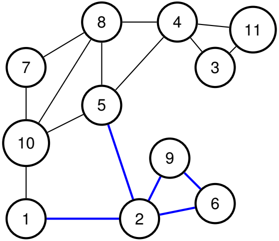
Paths, walks, cycles, tour.



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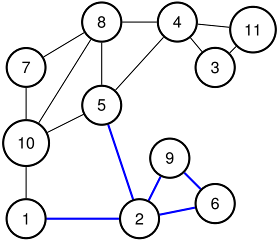
Walk is sequence of edges with possible repeated vertex or edge.

Paths, walks, cycles, tour.



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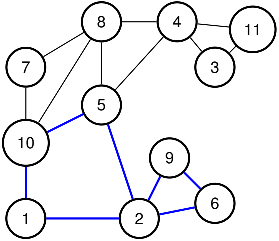

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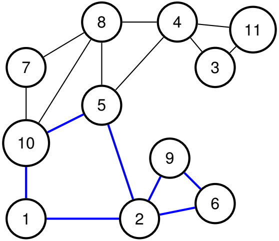
Paths, walks, cycles, tour.



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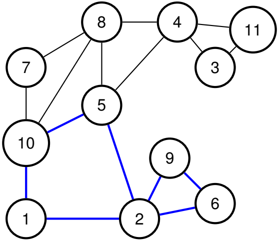

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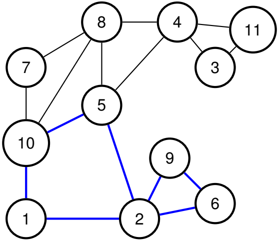
Paths, walks, cycles, tour.



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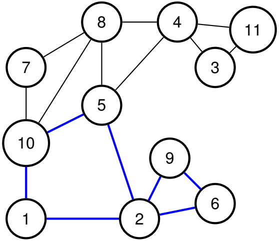
Quick Check!
Path is to Walk as Cycle is to ??

Paths, walks, cycles, tour.



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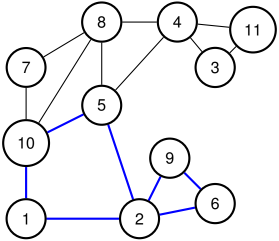
Path is to Walk as Cycle is to ?? Tour!

Paths, walks, cycles, tour.



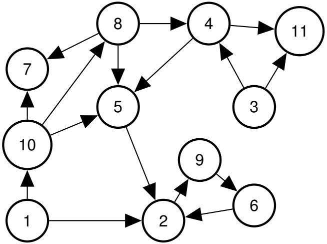
A path in a graph is a sequence of edges.

Path? $\{1,10\}$, $\{8,5\}$, $\{4,5\}$? No!
Path? $\{1,10\}$, $\{10,5\}$, $\{5,4\}$, $\{4,11\}$? Yes!

Path: $(v_1,v_2),(v_2,v_3),\ldots(v_{k-1},v_k)$.
Quick Check! Length of path? k vertices or $k-1$ edges.
Cycle: Path with $v_1 = v_k$. Length of cycle? $k-1$ vertices and edges!
Path is usually *simple*. No repeated vertex!
Walk is sequence of edges with possible repeated vertex or edge.
Tour is walk that starts and ends at the same node.

Quick Check!
Path is to Walk as Cycle is to ?? Tour!

Directed Paths.



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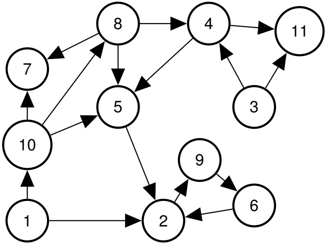

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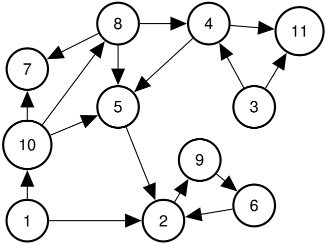

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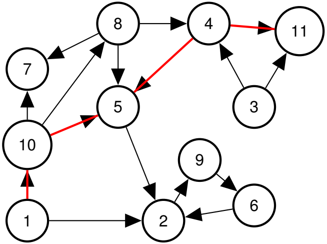

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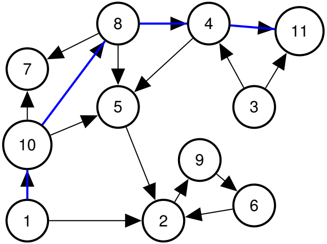

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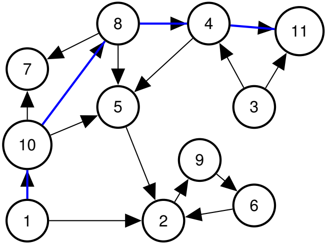

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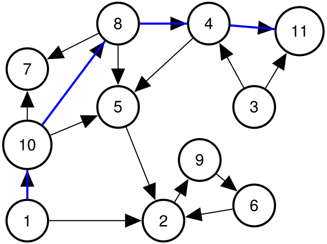

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Paths, walks,

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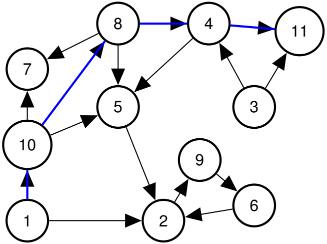

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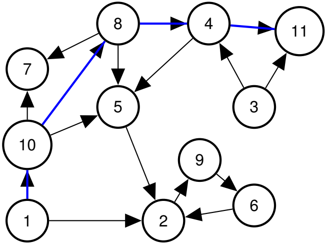

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Paths, walks, cycles, tours

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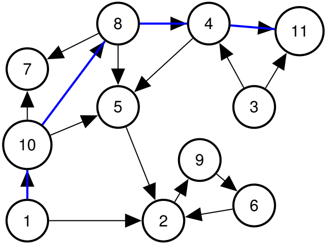

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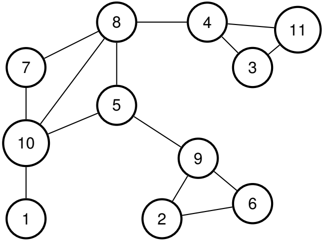
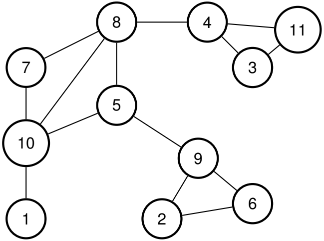
Paths, walks, cycles, tours ... are analogous to undirected now.

Connectivity



u and *v* are connected if there is a path between *u* and *v*.

Connectivity



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A connected graph is a graph where all pairs of vertices are connected.

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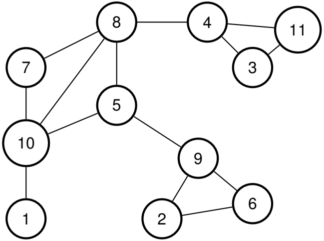

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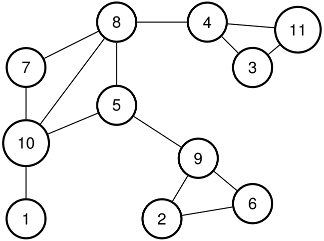

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Is graph connected?

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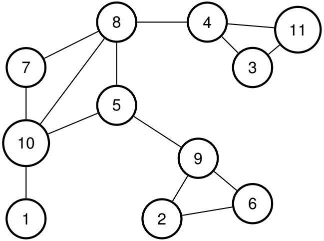

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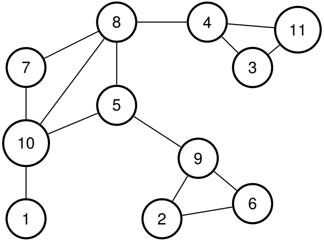

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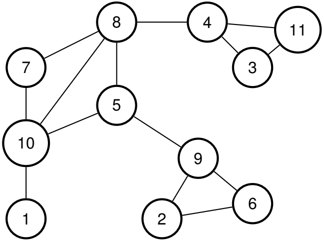

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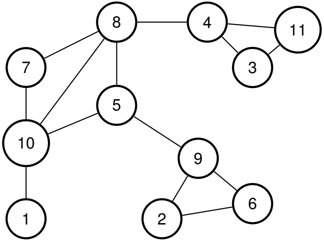
Proof:

Connectivity



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Proof: Use path from *u* to *x* and then from *x* to *v*.

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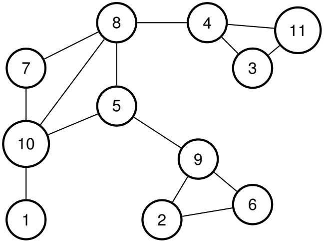

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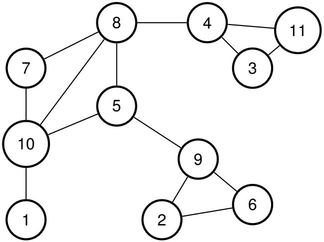

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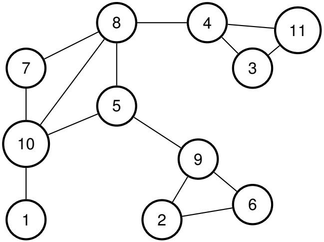

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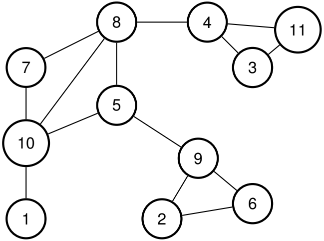

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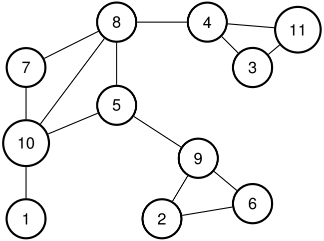

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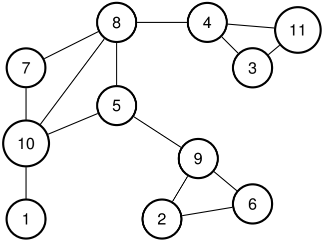
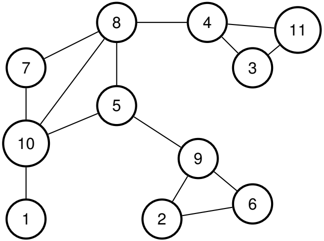


Is graph above connected?



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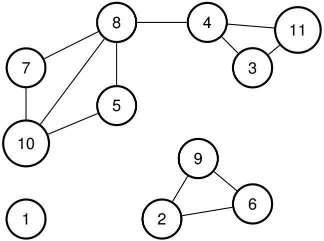
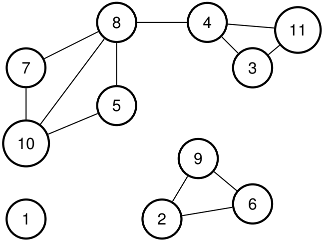


Is graph above connected? Yes!

How about now?



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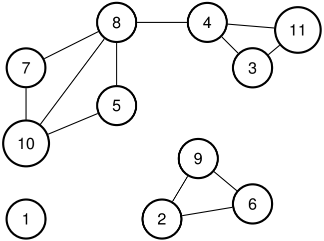

Is graph above connected? Yes!

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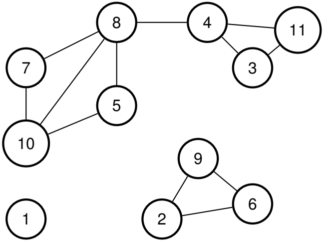
Connected Components?



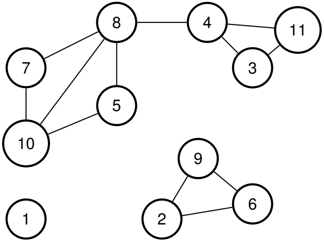
Is graph above connected? Yes!

How about now? No!

Connected Components? $\{1\}, \{10, 7, 5, 8, 4, 3, 11\}, \{2, 9, 6\}$.

A connected component is a maximal set of connected nodes in a graph.

Quick Check: Is $\{10, 7, 5\}$ a connected component?



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Finally..back to Euler!

An Eulerian Tour is a tour that visits each edge exactly once.

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Theorem: Any undirected graph has an Eulerian tour if and only if all vertices have even degree and is connected.
Proof of only if: Eulerian $\implies$ connected and all even degree.

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For starting node, tour leaves first ....then enters at end.

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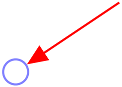

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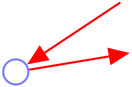

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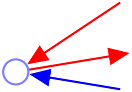

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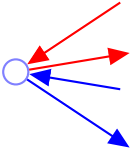

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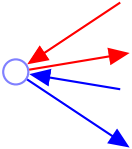

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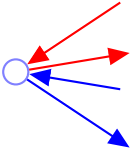

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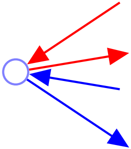

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For starting node, tour leaves first

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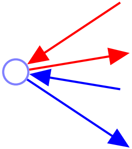

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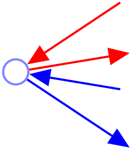

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Finding a tour!

Proof of if: Even + connected $\implies$ Eulerian Tour.
We will give an algorithm.

Finding a tour!

Proof of if: Even + connected $\Longrightarrow$ Eulerian Tour.
We will give an algorithm. First by picture.

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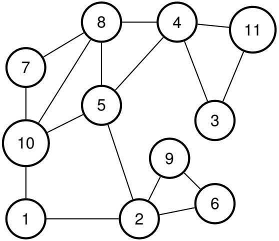

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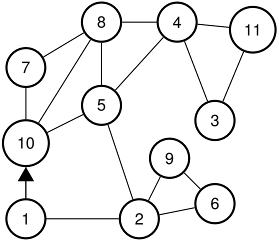

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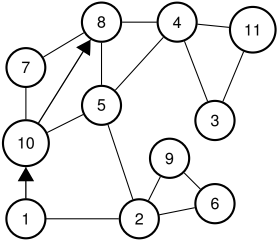

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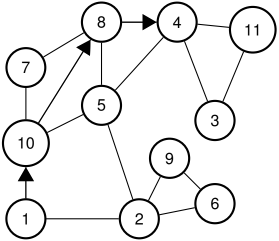

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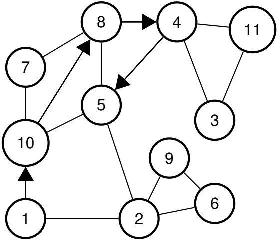

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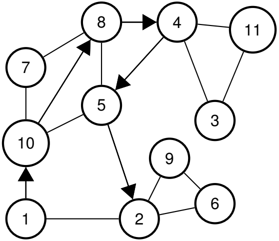

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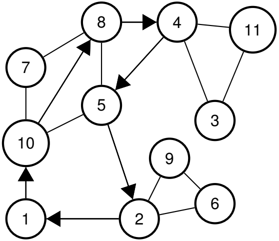

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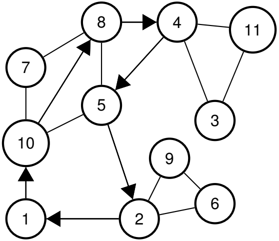

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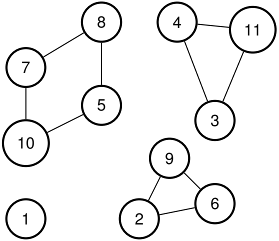

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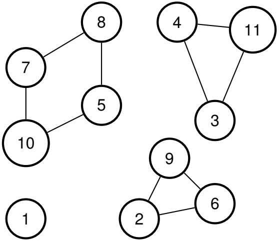

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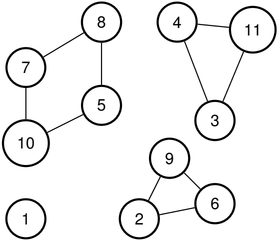

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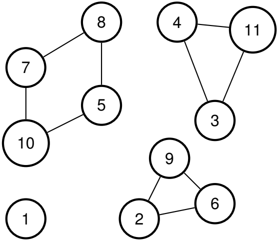

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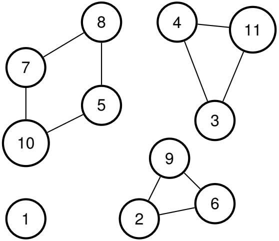

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Finding a tour!

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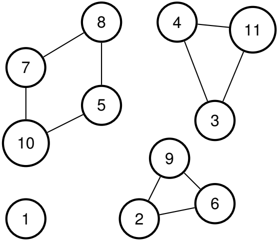

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   Example: $v_1 = 1$,

Finding a tour!

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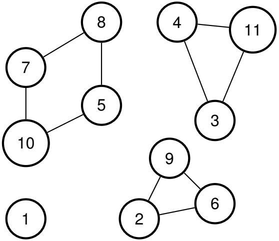

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   Example: $v_1 = 1$, $v_2 = 10$,

Finding a tour!

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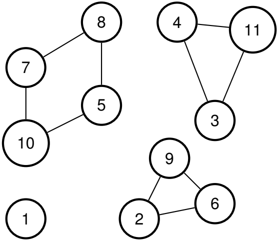

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   Why? G was connected.

   Let v_i be (first) node in G_i touched by C.

   Example: $v_1 = 1$, $v_2 = 10$, $v_3 = 4$,

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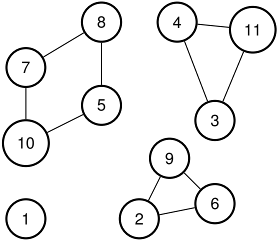

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Finding a tour!

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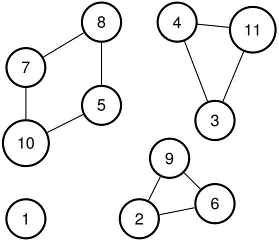

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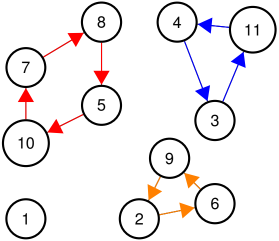

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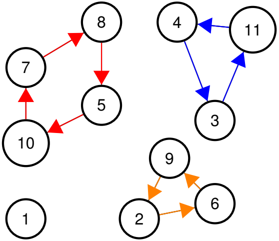

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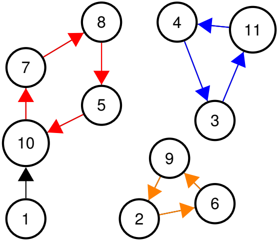

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   1,10

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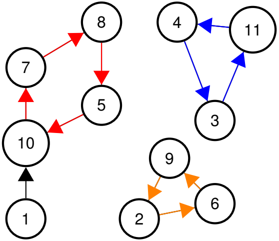

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   1,10,7,8,5,10

Finding a tour!

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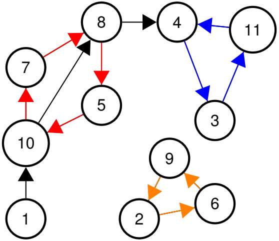

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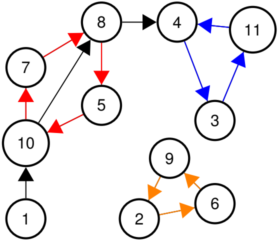

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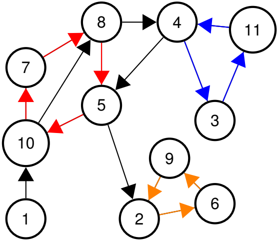

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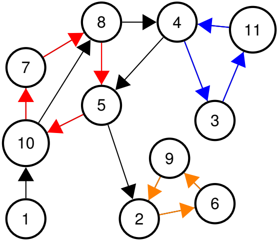

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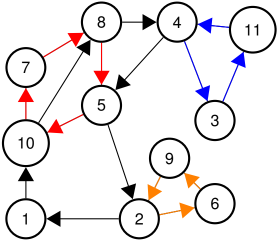

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   1,10,7,8,5,10 ,8,4,3,11,4 5,2,6,9,2 and to 1!

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

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Claim: Do get back to v!

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Claim: Do get back to v!
Proof of Claim: Even degree.

General case: Recursive algorithm, proof by induction.

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Claim: Do get back to v!

Proof of Claim: Even degree. If enter, can leave

General case: Recursive algorithm, proof by induction.

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Claim: Do get back to v!

Proof of Claim: Even degree. If enter, can leave except for v.

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Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

General case: Recursive algorithm, proof by induction.

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Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

General case: Recursive algorithm, proof by induction.

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Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

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Why is there a v_i in C?
G was connected $\implies$

General case: Recursive algorithm, proof by induction.

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Claim: Do get back to v!
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Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

General case: Recursive algorithm, proof by induction.

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Proof of Claim: Even degree. If enter, can leave except for v. $\square$

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Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\implies$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
Why is there a v_i in C?
G was connected $\Longrightarrow$
a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

2. Remove cycle, C, from G.

Resulting graph may be disconnected. (Removed edges!)

Let components be $G_1, \ldots, G_k$.

Let v_i be first vertex of C that is in G_i.
  Why is there a v_i in C?
    G was connected $\Longrightarrow$
      a vertex in G_i must be incident to a removed edge in C.

Claim: Each vertex in each G_i has even degree and is connected.
  Prf: Tour C has even incidences to any vertex v.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

Claim: Do get back to v!
Proof of Claim: Even degree. If enter, can leave except for v. □

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3. Find tour T_i of G_i

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3. Find tour T_i of G_i starting/ending at v_i.

General case: Recursive algorithm, proof by induction.

1. Take a walk from arbitrary node v, until you get back to v.

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3. Find tour T_i of G_i starting/ending at v_i.

4. Splice T_i into C where v_i first appears in C.

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Visits every edge once:
Visits edges in C

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Visits every edge once:
Visits edges in C exactly once.

General case: Recursive algorithm, proof by induction.

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4. Splice T_i into C where v_i first appears in C.

Visits every edge once:
Visits edges in C exactly once.
By induction for all other edges by induction on G_i.

General case: Recursive algorithm, proof by induction.

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