Today.

Polynomials.

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Secret Sharing.

A secret!

I have a secret!

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Example Applications:

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Cloud service backup: several vendors, each knows nothing.

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Example Applications:
Nuclear launch: need at least 3 out of 5 people to launch!
Cloud service backup: several vendors, each knows nothing.
data from any 2 to recover data.

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Share secret among n people.

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Polynomials

A **polynomial**

$$P(x) = a_d x^d + a_{d-1} x^{d-1} \cdots + a_0.$$

is specified by **coefficients** $a_d, \ldots a_0$.

[1] A field is a set of elements with addition and multiplication operations, with inverses. $GF(p) = (\{0, \ldots, p-1\}, + \pmod{p}, * \pmod{p})$.

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Polynomials $P(x)$ **with arithmetic modulo** p: [1] $a_i \in \{0, \ldots, p-1\}$ and

$$P(x) = a_d x^d + a_{d-1} x^{d-1} \cdots + a_0 \pmod{p},$$

for $x \in \{0, \ldots, p-1\}$.

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Polynomial: $P(x) = a_d x^4 + \cdots + a_0$

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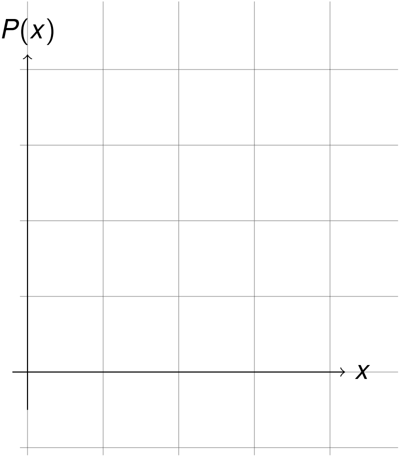

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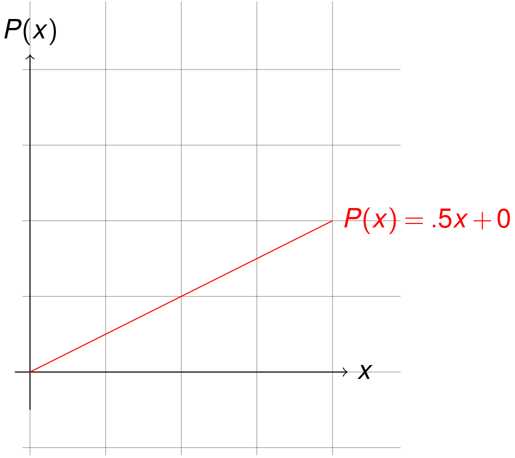

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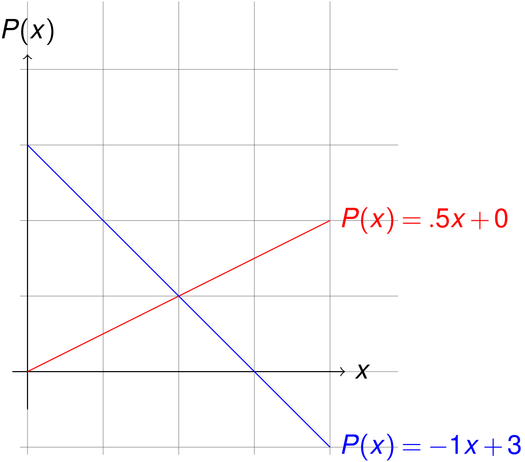

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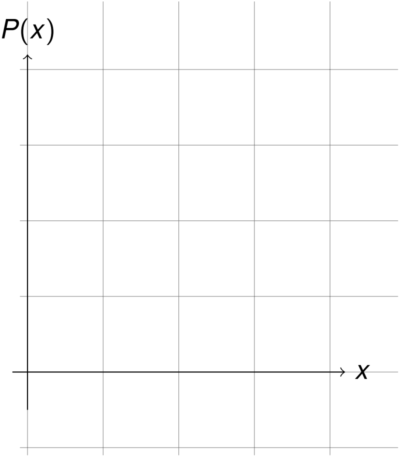

Parabola: $P(x) = a_2 x^2 + a_1 x + a_0$

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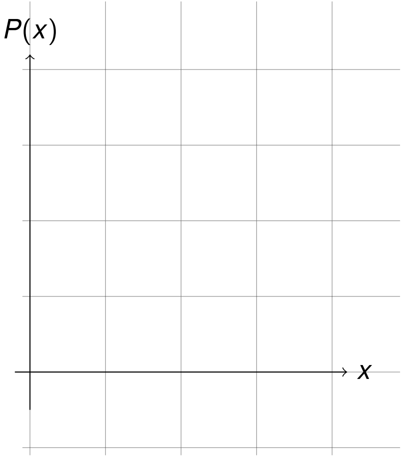

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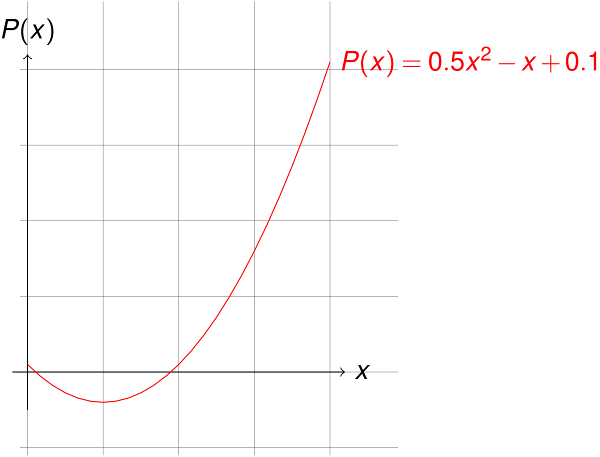

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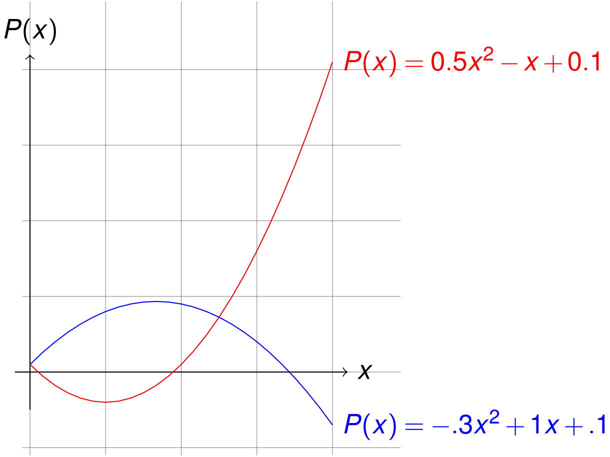

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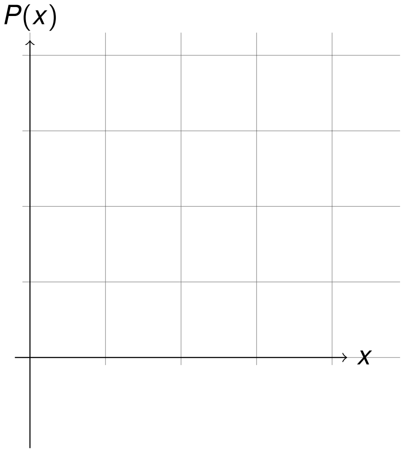

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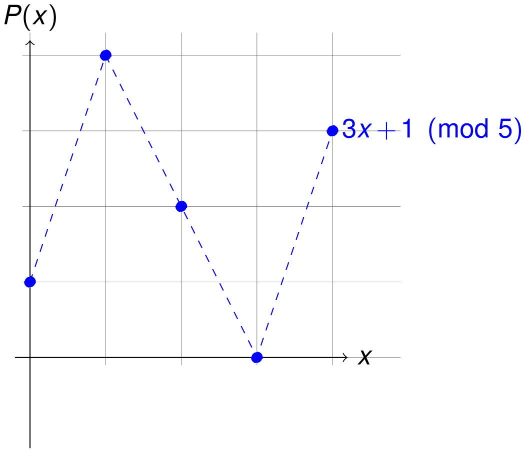

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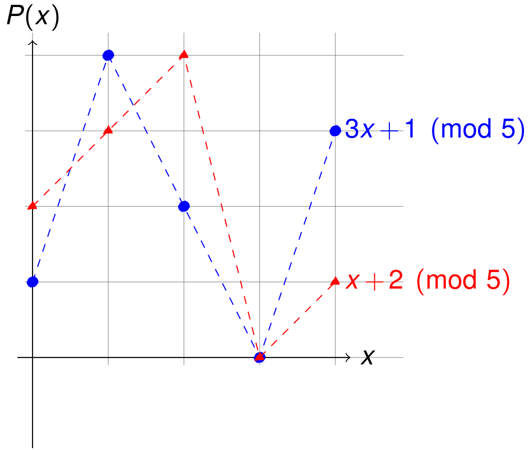

Finding an intersection.

$x + 2 \equiv 3x + 1 \pmod{5}$

$\implies 2x \equiv 1 \pmod{5}$

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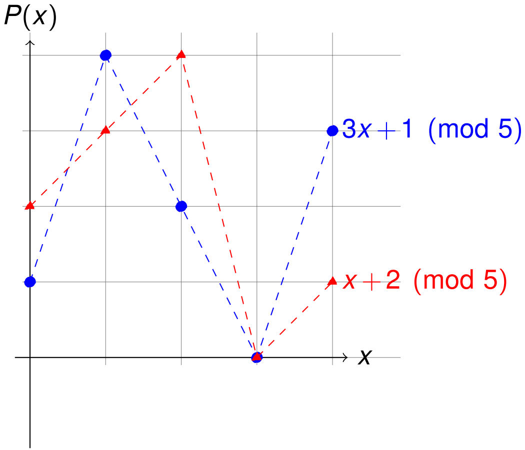

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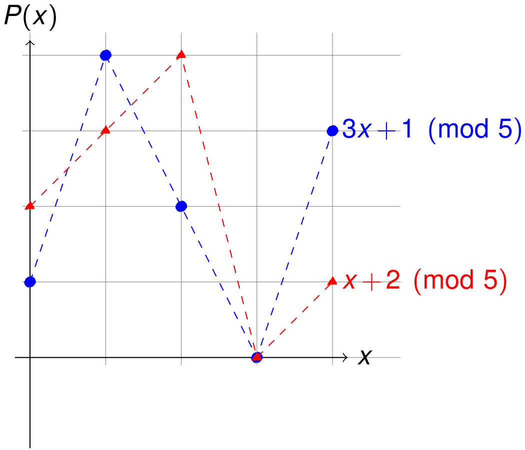

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Good when modulus is prime!!

Two points make a line.

Fact: Exactly 1 degree $\leq d$ polynomial contains $d+1$ points. [2]

[2] Points with different x values.

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Modular Arithmetic Fact: Exactly 1 degree $\leq d$ polynomial with arithmetic modulo prime p contains $d+1$ pts.

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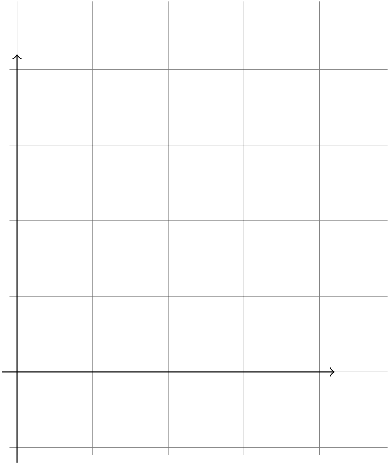
3 points determine a parabola.



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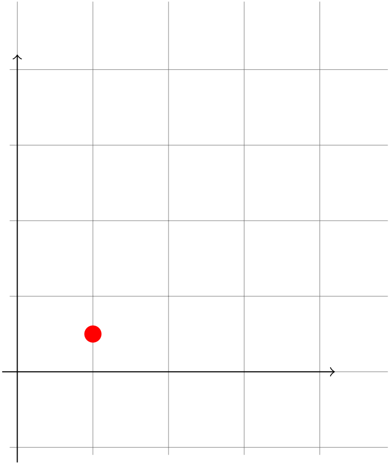

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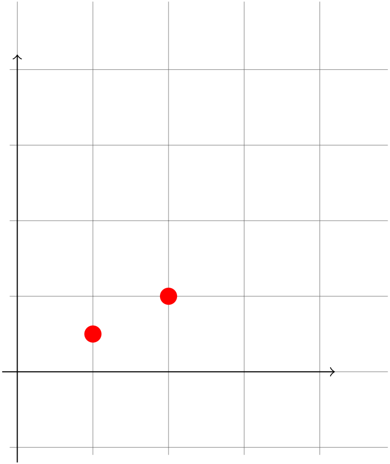

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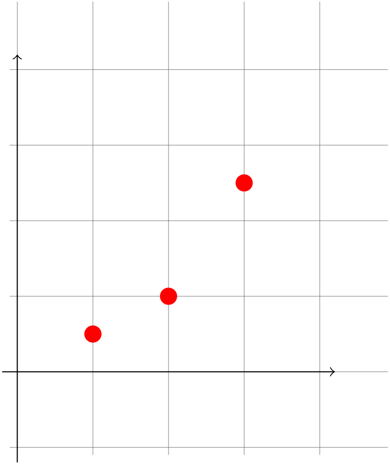

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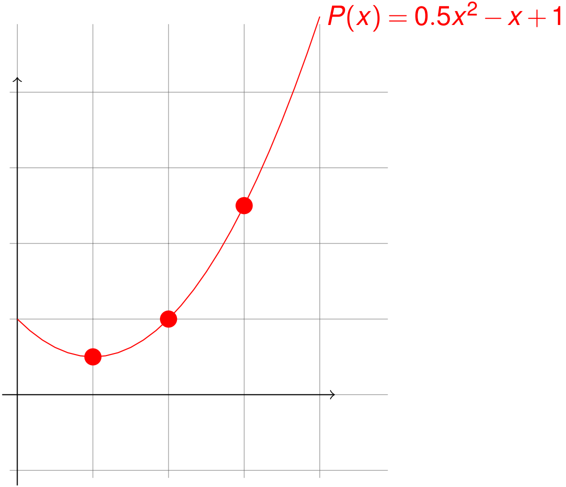

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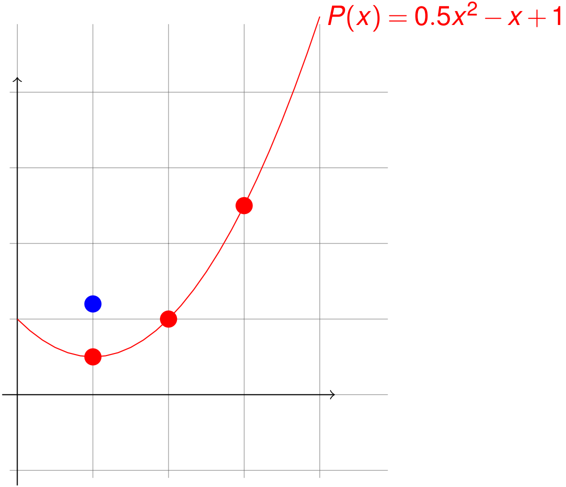

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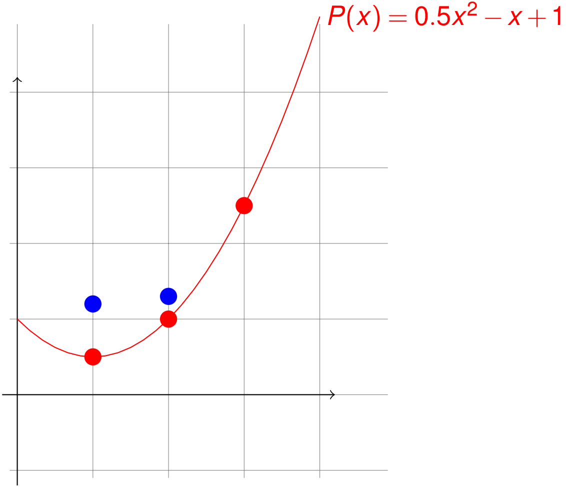

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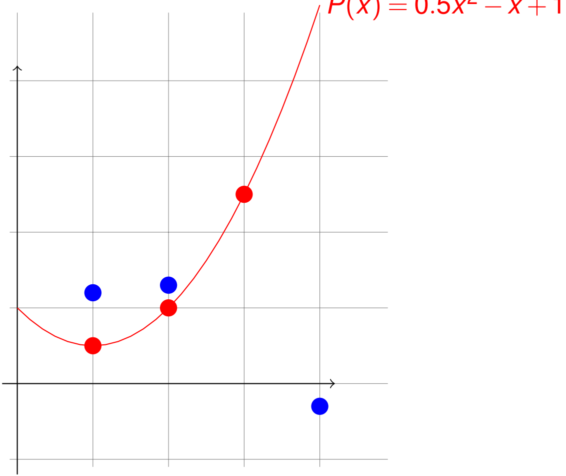

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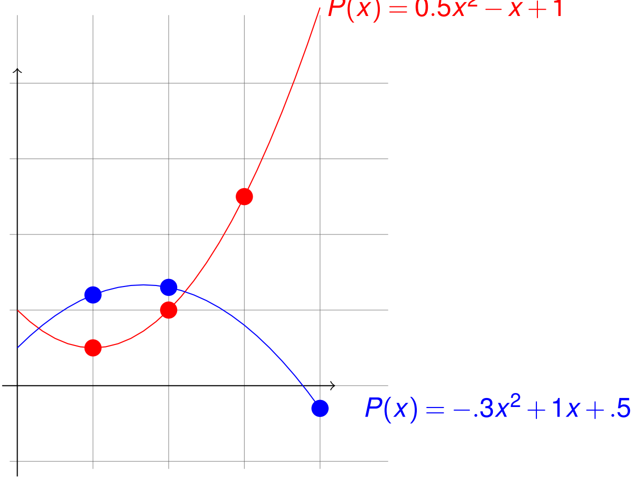

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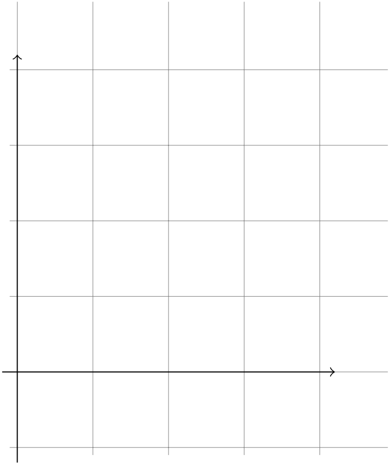

There is $P(x)$ contains blue points and *any* $(0, y)$!

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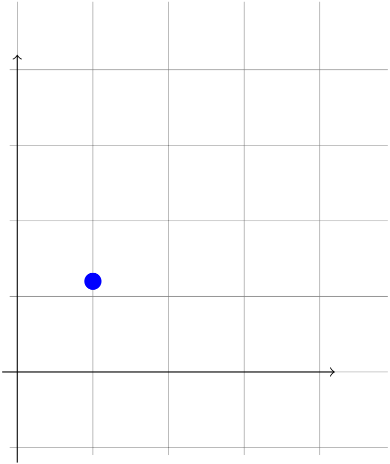

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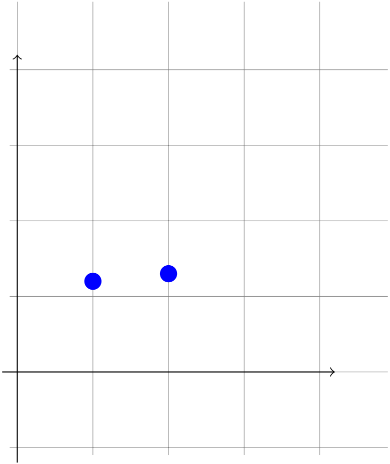

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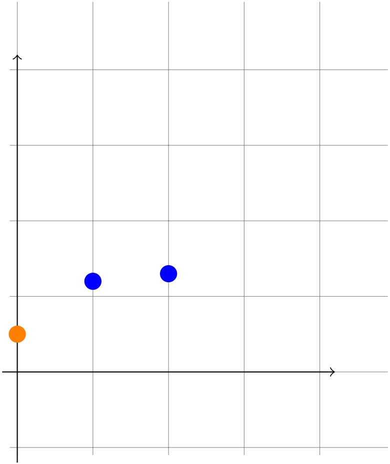

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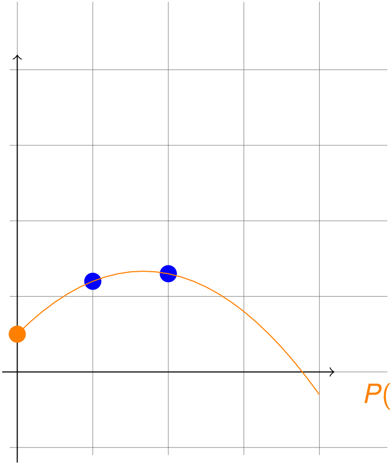

$P(x) = -.3x^2 + 1x + .5$

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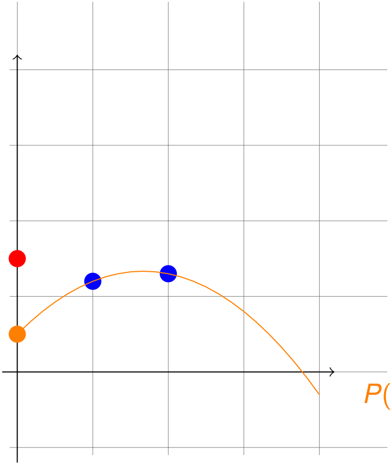

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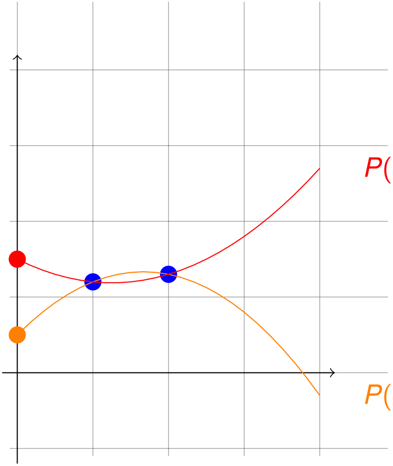

$P(x) = .2x^2 - .5x + 1.5$

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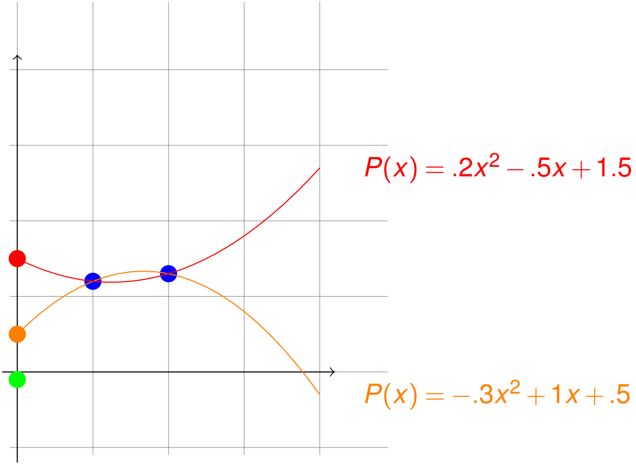

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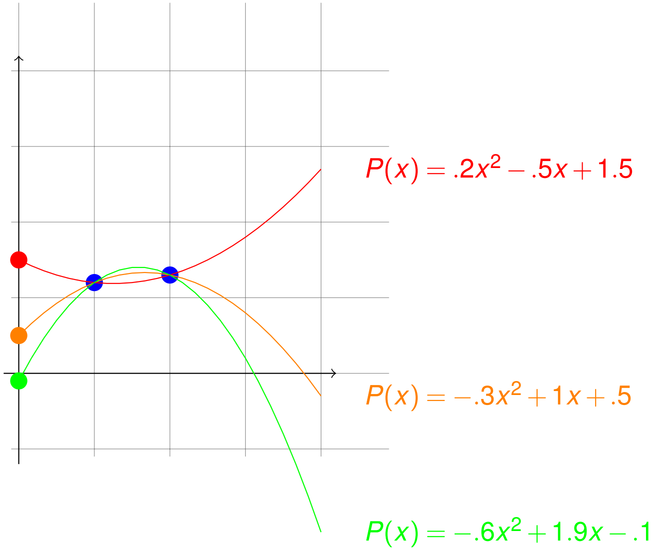

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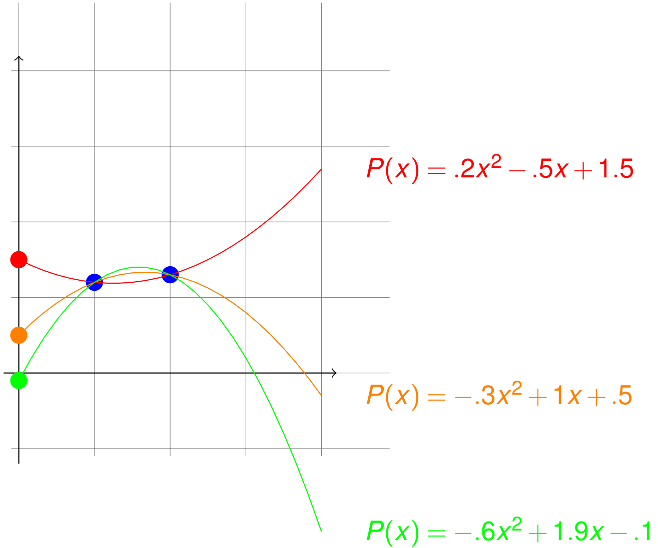

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Knowing k pts $\implies$ only one $P(x)$

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Modular Arithmetic Fact: Exactly 1 degree $\leq d$ polynomial with arithmetic modulo prime p contains $d+1$ pts.

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So polynomial is $2x^2 + 1x + 4 \pmod 5$

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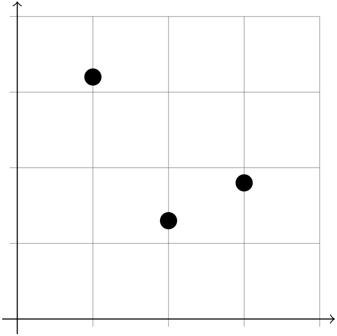
Construction proves the existence of a degree d polynomial!

Interpolation: in pictures.

Points: $(1, 3.2)$, $(2, 1.3)$, $(3, 1.8)$.



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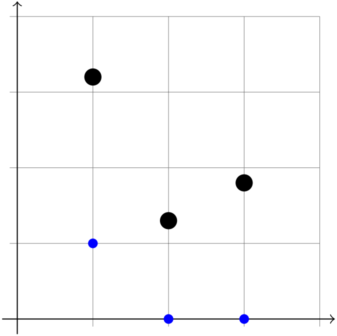

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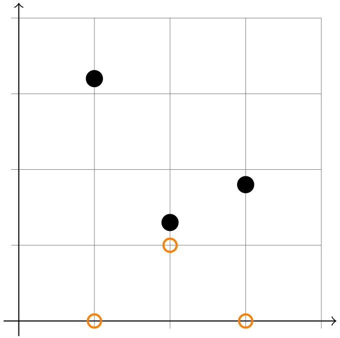

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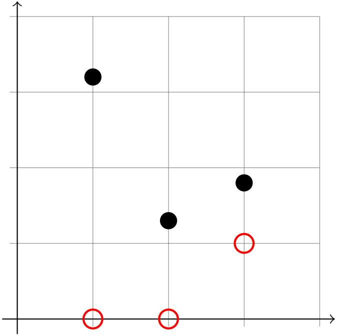

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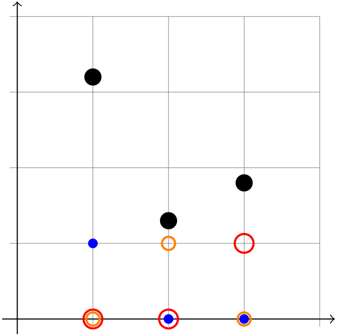

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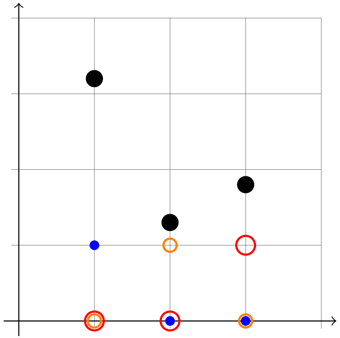

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Is it the only degree d polynomial that contains the points?

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Must prove **Roots fact.**

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That is, $P(x) = (x - a)Q(x) + r$

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$ab = 0 \implies a = 0$ or $b = 0$ in field.

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Roots fact: Any degree d polynomial has at most d roots.

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Intuitively, a field is a set with operations corresponding to addition, multiplication, and division.